# A
# COMPENDIOUS SYSTEM
# OF
# Natural Philoſophy:

With NOTES
Containing the
MATHEMATICAL DEMONSTRATIONS,
AND
Some Occaſional REMARKS.

## PART III.
## OPTICS.

To which is annexed a DISSERTATION on the Subject
of the Horizontal MOON.

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# A COMPENDIOUS SYSTEM OF Natural Philoſophy.

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## PART III.

## OPTICS.

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### CHAP. I.

### *Of the Nature and Propagation of* Light.

IN treating of the Nature of Fluids, I have explained ſuch *Phænomena* as reſult from ſmall Particles of Matter collected together, and acted upon according to the Laws of *Mechaniſm*; the Order of my Deſign now brings me to ſhew how according to the ſame Laws ſuch *Phænomena*, as reſult from the Emiſſion of infinitely ſmall Particles from luminous Bodies, are produced; which *Phænomena*, being the Means, whereby the Images of external Objects are repreſented to our Minds, by the Intervention of our Organs of

*Sight*, are for that Reaſon called *Optical*, and the Doctrine, by which they are explained, the Science of *Optics* *.

Every viſible Body emits or reflects inconceivably ſmall Particles of Matter from each Point of its Surface, which iſſue from it continually, (not unlike Sparks from a Coal) in ſtrait Lines and in all Directions. Theſe Particles entring the Eye, and ſtriking upon the *Retina* (a Nerve expanded on the back Part of the Eye to receive their Impulſes) excite in our Minds the Idea of Light. And as they differ in Subſtance, Denſity, Velocity, or Magnitude †, they produce in us the Ideas of different Colours; as will be explained in its proper Place.

That

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* Optics is generally divided into two Parts, *viz. Dioptrics*, under which is comprehended every Thing that relates to the Appearances of Bodies ſeen through tranſparent Subſtances; and *Catoptrics*, or what relates to the ſeeing of Bodies by reflected Light. To theſe we may add a third, which properly comes under neither of the former Diſtinctions, and that is, *the Doctrine of Colours*, which explains every Thing that relates to the Cauſes of the Diverſity of Colours obſervable in natural Bodies.

† 'Tis more probable, that they differ either in Magnitude, or Denſity, than in Velocity or Subſtance. For, if the Difference of Colours ariſe from the different Velocity of the Rays of Light, then the Colours of Objects would appear changed to an Eye placed under Water, or within any *Medium* differing from the Air in Denſity: For when a Ray of Light paſſes out of a *Medium* into another of different Denſity, it undergoes an Alteration in its Velocity, as will be explained hereafter. And to ſuppoſe them to differ in Subſtance, is contrary to that Uniformity of Things, which is obſervable in the Univerſe; as well as repugnant to that *Homogeneity* in the primo-

primo-

That the Particles, which conſtitute Light, are exceedingly ſmall, appears from hence, *viz.* that if a Hole be made through a Piece of Paper with a Needle, all the Rays of Light which proceed at the ſame Time, from all the Objects on one Side of it, are capable of paſſing through it at once without the leaſt Confuſion; for any one of thoſe Objects may as clearly be ſeen through it as if no Rays paſſed through it from any of the reſt. Further, if a Candle is lighted, and there be no Obſtacle in the Way to obſtruct the Progreſs of its Rays, it will fill all the Space within two Miles of it every Way with luminous Particles, before it has loſt the leaſt ſenſible Part of its Subſtance thereby.

That theſe Particles proceed from every Point of the Surface of a viſible Body, and in

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primogeneal Parts of Matter, which from the Experiments hitherto made, is thought to exiſt every where. Whereas, if we ſuppoſe them to differ either in Magnitude or Denſity, nothing is more eaſy than to ſee how thoſe of the ſame Kind ſhould, however refracted, produce the ſame Colours; and alſo how thoſe which produce different Colours, ſhould ſuffer different Degrees of Refraction in paſſing through the ſame *Medium*. As to the firſt, it is ſelf-evident, becauſe Refraction cannot alter their Magnitudes or Denſities; as to the ſecond, it is probable that the more intenſe and ſtronger Colours, the Rays of which ſuffer the leaſt Refraction, are produced by the larger, or more denſe Particles of Light: For, that ſuch Particles ſhould be leſs refracted than others, is quite conſonant to the Laws of Attraction of Coheſion, which, as it acts in Proportion to the Surfaces of Bodies only, muſt neceſſarily affect the larger, or the more denſe Particles, leſs than it does the reſt; becauſe ſuch have larger *Momenta* or Forces in Proportion to their Surfaces, than others have.

all Directions, is clear from hence, *viz*, becauſe where-ever a Spectator is placed with Regard to the Body, every Point of that Part of the Surface which is turned towards him, is viſible to him. That they proceed from the Body in right Lines, we are aſſured, becauſe juſt ſo many and no more will be intercepted in their Paſſage to any Place, by an interpoſed Object, as that Object ought to intercept, ſuppoſing them to come in ſuch Lines.

The Velocity, with which they proceed from the Surface of the viſible Body is no leſs ſurpriſing, than their Minuteneſs: The Method whereby Philoſophers eſtimate their Swiftneſs, is by Obſervations made on the Eclipſes of *Jupiter*'s Satellites, which Eclipſes to us appear about ſeven Minutes ſooner than they ought to do by Calculation, when the Earth is placed between the Sun and him, that is, when we are neareſt him, and as much later, when the Sun is between him and us, at which Time we are fartheſt from him; from whence it is concluded, that they require about ſeven Minutes to paſs over a Space equal to the Diſtance between the Sun and us, which is about eighty one Millions of Miles *.

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* This affords us another Proof of the ſurprizing Fineneſs of the Particles of Light; for the above-mentioned Velocity of the Rays is conſiderably more than a Million of Times greater than that of a Canon Ball. Were they not therefore inconceivably ſmall, the Eye would be rather wounded, than delighted with them; and the

A Stream of theſe Particles iſſuing from the Surface of a viſible Body in one and the ſame Direction, is called a *Ray of Light*.

As Rays proceed from a viſible Body in all Directions, they neceſſarily become thinner and thinner, continually ſpreading themſelves, as they paſs along, into a larger Space, and that in Proportion to the Squares of their Diſtances from the Body *; that is, at the Diſtance of two Spaces, they are four Times thinner, than they are at one; at the Diſtance of three Spaces, nine Times thinner, and ſo on: the Reaſon of which is, becauſe they ſpread themſelves in a twofold Manner, *viz.* upwards and downwards, as well as Side-Ways.

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the tender Flowers of Plants would be ſo far from being cheriſhed by them, that they would be torn in Pieces, and not able to ſtand before them.

* This Propoſition is demonſtrated mathematically thus; Let us conceive two concentric Surfaces ABD, and EFG (*Fig.* 1.) and in theſe, two ſimilar Portions ELFI, and AHBK; let the Rays CE and CF, with the reſt proceeding from the Center C, fall upon the Portion ELFI and cover it; 'tis evident from Inſpection of the Figure, that the ſame Rays at the Diſtance CH will cover the Portion AHBK only; now theſe Rays being the ſame in Number at each Place, will be thinner in the former, than they are in the latter, in Proportion as that is larger than this; but theſe Spaces being ſimilar Portions of the Surfaces of Spheres, bear the ſame Proportion to each other, that the Surfaces themſelves do, that is, they are to each other as the Squares of their *Radii* CL, CH; the Rays therefore are more diffuſed, or thinner in Proportion to the Squares of the ſame *Radii*, or of their Diſtances from the luminous Point C. *Q. E. D.*

The Particles of Light are ſubject to the Laws of Attraction of Coheſion like other ſmall Bodies, for if a Ray of Light be made to paſs by the Edge of a Knife, it will be diverted from its natural Courſe, and be inflected towards the Edge of the Knife. The like Inflection happens to a Ray when it enters obliquely into a denſer or rarer Subſtance than that in which it was before, in which Caſe it is ſaid to be *refracted*; the Laws of which Refraction are the Subject of the following Chapter *.

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* The *Carteſian* Notion of Light, was not, that it is propagated from luminous Bodies by the Emiſſion of ſmall Particles, but that it was communicated to the Organ of Sight by their Preſſure upon the *Materia ſubtilis*, with which they ſuppoſed the Univerſe to be full. But according to this *Hypotheſis*, it could never be dark; becauſe when a Fluid ſuſtains any Preſſure, if that Fluid fills all the Space it takes up, abſolutely, without leaving any Pores, which is the Caſe of the ſuppoſed *Materia ſubtilis*; then that Preſſure muſt neceſſarily be communicated *equally* and *inſtantaneouſly* to every Part: And therefore, whether the Sun were above or below the Horizon, the Preſſure communicated, and conſequently the Light, would be the ſame. And farther, as the Preſſure would be inſtantaneous, ſo would the Light, which is contrary to what is collected, as we obſerved above, from the Eclipſes of *Jupiter's* Satellites.

CHAP.

## CHAP. II.

## *Of the Cauſe of* Refraction, *and the* Law *by which it is performed.*

WHatever Subſtance a Ray of Light paſſes through, or if it paſs through a Space void of all Subſtance, it is ſaid by Philoſophers to paſs through a *Medium*; and therefore if it paſſes out of any Subſtance, as Air or Glaſs, into a *Vacuum*, it is ſaid to paſs out of one *Medium* into another.

All Bodies being endued with an attractive Force, which is extended to ſome Diſtance beyond their Surfaces; when a Ray of Light paſſes out of a rarer into a denſer *Medium* (if this latter has a greater attractive Force than the former, as is commonly the Caſe *, and what we ſhall hereafter always ſuppoſe, unleſs it be mentioned to the contrary) the Ray juſt before its Entrance, will begin to be attracted towards the denſer *Medium*, and this Attraction will continue to act upon it, till ſome Time after it has entered the *Medium*, as we ſhall ſhew by and by; and therefore if a Ray approaches a denſer *Medium* in a Direction perpendicular to its Surface, its Velocity will be continually accelerated during its Paſſage through the Space in which that Attraction exerts itſelf; and therefore, after it has paſſed that Space, it will move on, till it arrives at the oppoſite Side of the *Medium*, with a greater Degree of Velocity than it had before it entered. So that in this Caſe its Velocity only will be altered. Whereas, if a Ray enters a denſer *Medium* obliquely, it will not only have its Velocity augmented thereby, but its Direction will become leſs oblique to the Surface. Juſt as when a Stone is thrown downwards obliquely from a Precipice, it falls to the Surface of the Ground in a Direction nearer to a perpendicular one, than that with which it was thrown from the Hand. From hence we ſay a Ray of Light in paſſing out of a rarer into a denſer *Medium*, is refracted *towards the Perpendicular*; that is, ſuppoſing a Line drawn perpendicularly to the Surface of the *Medium*, through the Point where the Ray enters, and extended both Ways, the Ray in paſſing through the Surface is refracted or bent towards the perpendicular Line; or, which is the ſame Thing, the Line which it deſcribes by its Motion after it has paſſed through the Surface, makes a leſs Angle with the Perpendicular, than the Line it deſcribed before. All which may be illuſtrated in the following Manner.

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* In oily and inflammable Bodies it happens otherwiſe; for they are obſerved to attract more ſtrongly than others of greater Denſity.

Let us ſuppoſe firſt, that the Ray paſſes out of a *Vacuum* into the denſer *Medium* ABCD, (*Fig.* 2.) and that the attractive Force of each Particle in the *Medium* is extended from its reſpective Center to a Diſtance equal to that which is between the Lines AB and EF, or AB and GH; and let KL be the Path deſcribed by a Ray of Light in its Progreſs towards the denſer *Medium*. This Ray when it arrives at L will enter the attractive Forces of thoſe Particles which lie in AB the Surface of the denſer *Medium*, and will therefore ceaſe to proceed any longer in the right Line KLM, but will be diverted from its Courſe by being attracted towards the Line AB, and will begin to deſcribe the Curve LN, paſſing through the Surface AB in ſome new Direction as OQ, thereby making a leſs Angle with a Line as PR drawn perpendicularly through the Point N, than it would have done, had it proceeded in its firſt Direction KLM.

Farther, whereas we have ſuppoſed the attractive Force of each Particle to be extended through a Space equal to the Diſtance between AB and EF, it is evident the Ray after it has entered the Surface, will ſtill be attracted downwards, till it has arrived at the Line EF; for till that Time, there will not be ſo many Particles above it which will attract it upwards, as below, that will to attract it downwards. So that after it has entered the Surface at N, in the Direction OQ, it will not proceed in that Direction, but will continue to deſcribe a Curve, as N S, after which it will proceed ſtrait on towards the oppoſite Side of the *Medium*, being attracted equally every Way; and therefore will at laſt proceed in the Direction XST ſtill nearer the Perpendicular PR than before.

Now if we ſuppoſe the Space ABYZ not to be a *Vacuum*, but a rarer *Medium* than the other, the Caſe will ſtill be the ſame; but the Ray will not be ſo much refracted from its rectilineal Courſe, becauſe the Attraction of the Particles of the upper *Medium* being in a contrary Direction to that of the Attraction of thoſe in the lower one, the Attraction of the denſer *Medium* will in ſome Meaſure be deſtroyed by that of the rarer.

On the contrary, when a Ray paſſes out of a denſer into a rarer *Medium*, if its Direction be perpendicular to the Surface of the *Medium*, it will only loſe ſomewhat of its Velocity, in paſſing through the Space of Attraction of that *Medium* (that is, the Space wherein it is attracted more one Way than it is another). If its Direction be oblique, it will continually recede from the Perpendicular during its Paſſage, and by that Means have its Obliquity encreaſed, juſt as a Stone thrown up obliquely from the Surface of the Earth increaſes its Obliquity all the time it riſes. Thus, ſuppoſing the Ray TS paſſing out of the denſer *Medium* ABCD into the rarer ABYZ, when it arrives at S it will begin to be attracted downwards, and ſo will deſcribe the Curve SNL, and then proceed in the right Line LK, making a larger Angle with the Perpendicular PR, than the Line TSX in which it proceeded during its Paſſage through the other *Medium*.

The Space through which the Attraction of Coheſion of the Particles of Matter is extended is ſo very ſmall, that in conſidering the Progreſs of a Ray of Light out of one *Medium* into another, the Curvature it deſcribes in paſſing through the Space of Attraction is generally neglected; and its Path is ſuppoſed to be bent, or in the uſual Terms, the Ray is ſuppoſed to be refracted only in the Point where it enters the denſer *Medium*.

Now the Line, which a Ray deſcribes before it enters a denſer or a rarer *Medium* is called the *Incident Ray*; that which it deſcribes after it has entered, is the *Refracted Ray*.

The Angle comprehended between the Incident Ray and the Perpendicular, is the *Angle of Incidence*; and that between the refracted Ray and the Perpendicular, is the *Angle of Refraction*.

There is a certain and immutable Law or Rule, by which Refraction is always performed; and that is this; Whatever Inclination a Ray of Light has to the Surface of any *Medi-*

*um* before it enters it, the Degree of Refraction will always be ſuch, that the Proportion between the Sine of the Angle of its Incidence, and that of the Angle of its Refraction, will always be the ſame in that *Medium* *.

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* *Lemma.* If from a Point as M (*Fig.* 4.) taken any where without the Circle PNQ, a Line as MP be drawn paſſing through L the Center of the Circle, and terminated in the Circumference at P, the Product of MQ multiplied by MP is equal to the Difference between the Squares of ML and PL.

*Dem.* Call MQ, $a$; and the Radius of the Circle LQ or LP, $b$; then will the Diameter QP be expreſſible by $2b$, and the whole Line MP, by $a+2b$; then multiplying MQ by MP, that is, $a$ by $a+2b$, we have for the Product of this, $aa+2ab$. Now the Square of the Line ML, which is expreſſible by $a+b$, is $aa+2ab+bb$, and the Square of PL is $bb$; but the Difference between theſe Squares, *viz.* $aa+2ab+bb$ and $bb$ is evidently $aa+2ab$; and therefore the Product of MQ multiplied by MP is equal to the Difference between the Squares of ML and PL. *Q. E. D.*

*Dem.* When a Ray of Light paſſes through the Space of Attraction of any *Medium*, it is evident that its Motion will be ſubject to the like Laws with that of *Projectiles*, provided we ſuppoſe it to be acted upon with an equal Degree of Force during its whole Paſſage through that Space, as is the Caſe of *Projectiles* to whatever Height they are thrown from the Earth. We will therefore put a Caſe as nearly parallel as may be to that which was demonſtrated of *Projectiles* in the ſeventh Chapter of the firſt Part; and ſuppoſe firſt, that the Force of Attraction of the denſer *Medium* is at all Diſtances the ſame as far as it reaches, and that the Ray proceeds out of a denſer into a rarer *Medium*; in which Caſe it will be attracted back towards the denſer *Medium*, during its Paſſage through the Space of Attraction, in like Manner as a *Projectile* thrown upwards is while it riſes from the Earth. Let then ABCD (*Fig.* 4.) repreſent the denſer *Medium*, and ABEF the Space of Attraction; and let GH be a Ray about to enter the Force of Attraction at H, and let GH be produced to M. Now 'tis evident, that in this Suppoſition, the Ray when at H, is in the ſame Circumſtances with a *Projectile* about to be thrown upwards from H towards M; it will therefore deſcribe a Portion of a *Parabola*

as

To illuſtrate this, Let us ſuppoſe ABCD (*Fig.* 3.) to repreſent a rarer, and ABEF a denſer *Medium*; let GH be a Ray of Light paſſing

as HI; to which the Line HM will be a Tangent at H; and the Line IK, in which it would proceed after it has paſſed the Space of Attraction, a Tangent to it at I, for after having left the attractive Force at I, it goes ſtrait on in its laſt Direction. Let the Perpendicular IR be drawn meeting GH produced in M, and let KI be produced to L. On the Center L with the Radius LI, deſcribe the Circle PNQ, let fall the Perpendicular LO upon MR, and join the Points L and N. Now it is demonſtrated in the Caſe of Projectiles, that the *Parameter* of the Point H is equal to $\frac{HMq}{MI}$, and therefore the *Parameter* multiplied by MI is equal to HMq. And it is there farther demonſtrated, that the ſaid *Parameter* is equal to a fourth Part of the Height which a Body muſt fall from, to acquire the Velocity the Projectile has at H; this Parameter therefore is a Quantity not at all depending on the Direction of the Projectile, but its Velocity only; and conſequently in the preſent Suppoſition it is a given Quantity, the Ray GH being ſuppoſed to have the ſame Velocity, whatever is its Inclination to the Surface AB. Now the Tangent KI being produced to L, will by the Property of the *Parabola*, biſect the other Tangent HM, wherefore the Line LQ being parallel to HR, MR will alſo be biſected in O; and adding the equal Lines OI and ON to each Part, MN will be equal to IR; but the Line IR is alſo a Line independent of the Inclination of the Ray GH, its Length being determined by the Breadth of the Space of Attraction ABEF only, and therefore MN is a given Quantity. Now, whereas MI, when multiplied by the Parameter of the Point H, which before was ſhewn to be a given Line, is equal to the Square of HM, therefore the ſame Line MI when multiplied by any other given Line, (*viz.* MN) if it is not equal to, will nevertheleſs bear a given Proportion to the Square of HM: But ſince MI multiplied by MN bears a given Proportion (*viz.* a Proportion that does not depend on the Inclination of the Ray GH) to the Square of MH, its equal, *viz.* the Product of MQ multiplied by MP (37. El. 3.) or what is equal to this, the Difference between the Squares of ML and PL (by the foregoing *Lemma*) or, which is the ſame Thing, of ML and LI, (becauſe PL and LI are *Radii* of the ſame Circle) paſſing through the firſt and entering the ſecond at H, and let HI be the refracted Ray; then ſuppoſing the Perpendicular PR drawn through the Point H, on the Center H, and with any

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Circle) does ſo too. Now the Square of ML bears alſo a given Proportion to the Square of MH (ML being equal to half MH) conſequently there is a given Proportion between the Square of ML and the Difference of the Squares of ML and LI; and therefore there is a certain Proportion between the Lines themſelves, *viz.* between ML and LI. But in every Triangle the Sides are proportionable to the Sines of their oppoſite Angles, therefore in the Triangle MLI, the Sine of the Angle LMI has a given Proportion to the Sine of the Angle LIM, or of its Complement to two right ones MIK (for they have the ſame Sine): But LMI being an Angle made by the incident Ray AH produced, with the Perpendicular RM, is the Angle of Incidence, and MIK being made by the refracted Ray IK, and the ſame Perpendicular, is the Angle of Refraction, therefore in this Caſe there is a conſtant *Ratio* between the Sine of the Angle of Incidence, and that of the Angle of Refraction. *Q. E. D.*

We have in the foregoing Demonſtration ſuppoſed that the Force of Attraction is every where uniform, but if it be otherwiſe, provided it be the ſame every where at the ſame Diſtances from the Surface AB, the Proportion between the forementioned Sines will ſtill be a given one. For, let us imagine the Space of Attraction divided into parallel Planes, and the Attraction to be the ſame through the whole Breadth of each Plane though different in different Planes, the Sine of the Angle of Incidence out of each will, by what has been demonſtrated above, be to the Sine of the Angle of Refraction into the next in a given *Ratio*; and therefore, ſince the Sine of the Angle of Refraction out of one will be the Sine of the Angle of Incidence into the next, it is evident that the Sine of the Angle of Incidence into the firſt will be in a given *Ratio* to the Sine of the Angle of Refraction out of the laſt. Now let us ſuppoſe the Thickneſs of theſe Planes diminiſhed *in infinitum*, and their Number proportionably increaſed, the Law of Refraction will ſtill continue the ſame; and therefore whether the Attraction be uniform or not, there will be a conſtant *Ratio* between the Sine of the Angle of Incidence and of Refraction. *Q. E. D.*

*Radius* deſcribe the Circle APBR, and from G and I where the incident and refracted Rays cut the Circle, let fall the Lines GK and IL perpendicularly upon the Line PR, the former of theſe will be the Sine of the Angle of Incidence, the latter of Refraction. Now if in this Caſe, the Ray GH is ſo refracted at H, that GK is double or treble, *&c.* of IL, then whatever other Inclination the Ray GH might have had, the Sine of its Angle of Incidence would have been double, or treble, *&c.* to that of its Angle of Refraction. For Inſtance, had the Ray paſſed in the Line MH before Refraction, it would have paſſed in ſome Line, as HN afterwards, ſo ſituated that MO ſhould have been double or treble, *&c.* of NQ.

When a Ray paſſes out of a *Vacuum* into Air, the Sine of the Angle of Incidence is found to be to that of Refraction, as 100036 to 100000.

When it paſſes out of Air into Water, as about 4 to 3.

When out of Air into Glaſs, as about 17 to 11.

When out of Air into a Diamond, as about 5 to 2.

## CHAP. III.

## *Of the Refraction of* Light *in paſſing thro' plain and ſpherical* Surfaces.

AS Rays of Light are capable of having their Progreſs altered by Refraction or Reflection, it is poſſible they may have various Inclinations and Directions different from thoſe which they naturally acquire by proceeding from the Surfaces of viſible Bodies.

When they recede from each other as they paſs along, they are ſaid to *diverge*; and the Point they proceed from, is called the *Radiant Point*.

When they proceed towards any Point approaching nearer together in their Progreſs, they are then ſaid to *converge*; and the Point towards which they tend, is called the *Focus*.

This *Focus* may be either *real* or *imaginary*; it is ſaid to be real, when the Rays actually proceed to it; but if they are intercepted in their Progreſs, or turned another Way before they reach it, it is called their *imaginary Focus*.

Sometimes it happens, that Rays are ſo refracted or reflected, that they proceed afterwards, as from ſome Point, which is not their true Radiant, then alſo that Point is called their *imaginary Focus*.

When they proceed in parallel Lines, they are then called *parallel Rays*; and both their Focus and radiant Point is ſuppoſed to be at an infinite Diſtance.

When Rays paſs out of one *Medium* into another, they ſuffer various Alterations in their Motion. All which are expreſſed in the eighteen following Propoſitions.

I. When *parallel* Rays fall obliquely on a plain Surface of a *Medium* of different Denſity, they are *parallel* alſo after Refraction. For having all the ſame Inclination to the Surface, they ſuffer an equal Degree of Refraction.

II. When diverging Rays paſs out of a rarer into a denſer *Medium* through a plain Surface, they are made thereby to diverge leſs.

For being all refracted towards their reſpective Perpendiculars, (which Perpendiculars are parallel to one another) they are brought nearer to a Paralelifm themſelves, that is, they are made to diverge leſs than before.

*See this and the following Caſes expreſſed more determinately, and demonſtrated in the Note below* *.

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* I. When Rays paſs out of one *Medium* into another of different Denſity through a plain Surface, if they diverge, the focal Diſtance will be to that of the radiant Point; if they converge, it will be to that of the imaginary Focus of the incident Rays, as the

III. When they proceed out of a denſer into a rarer *Medium*, the contrary happens; for then being refracted from their reſpective Perpendiculars, they are made to diverge more.

IV. So

the Sine of the Angle of Incidence is to that of the Angle of Refraction.

This Propoſition admits of four Caſes.

*Caſe* 1. Of diverging Rays paſſing out of a rarer into a denſer *Medium*.

*Dem.* Let X (*Fig.* 10.) repreſent a rarer, and Z a denſer *Medium*, ſeparated from each other by the plain Surface AB; ſuppoſe CE and CD two diverging Rays proceeding from the Point C, the one perpendicular to the Surface, the other oblique; through E draw the Perpendicular PK. The Ray CD being perpendicular to the Surface will proceed on in the right Line CQ, but the other falling obliquely on it at E, and there entring a denſer *Medium*, will ſuffer a Refraction towards the Perpendicular EK. Let then EG be the refracted Ray, and produce it back till it interſects DC produced alſo, in F; this will be the focal Point. On the Center E and with the Radius EF, deſcribe the Circle A F B Q, and produce EC to H; draw HI the Sine of the Angle of Incidence, and GK that of Refraction; equal to this is FP or CM which let be drawn. Now if we ſuppoſe the Points D and E contiguous, or nearly ſo, then will the Line HE be almoſt coincident with FD, and therefore FD will be to CD as HE to CE; but HE is to CE as HI to CM, becauſe the Triangles HIE and CME are ſimilar, that is, the focal Diſtance of the Ray CE is to the Diſtance of the Radiant Point, as the Sine of the Angle of Incidence is to that of the Angle of Refraction. *Q. E. D.*

Obſ. 1. *Whereas the* Ratio *of* IE *to* ME, *or which is the ſame Thing, that of* nD *to* CD *bears the exact Proportion of* HI *to* CM; *and becauſe this, (being the* Ratio *of the Sine of the Angle of Incidence to that of the Angle of Refraction) is always the ſame, the Line* In *is in all Inclinations of the Ray* CE, *at the ſame Diſtance from* CM; *conſequently had* CE *been coincident with* CD, *the Point* H *had fallen upon* n: *and becauſe the Circle paſſes through both* H *and* F, F *would alſo have fallen upon* n; *upon*

*which*

IV. So when converging Rays paſs out of a rarer into a denſer *Medium*, through a plain Surface, they are made thereby to converge leſs.

For

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*which Account the Focus of the Ray* CE *would have been there. But the Ray* CE *being oblique to the Surface* DB, *the Point* H *is at ſome Diſtance from* n; *and therefore the Point* F *is neceſſarily ſo too, and the more ſo by how much the greater that Diſtance is: from whence it is clear, that no two Rays flowing from the radiant Point* C *and falling with different* Obliquities *on the Surface* BD, *will after Refraction there, proceed as from the ſame Point; therefore ſtrictly ſpeaking, there is no one Point in the Line* D *produced, that can more properly be called the Focus of Rays flowing from* C, *than another: for thoſe which enter the refracting Surface near* D, *will after Refraction proceed, as has been obſerved, from the Parts about* n; *thoſe which enter near* E, *will flow as from the Parts about* F; *thoſe which enter about* T, *as from ſome Points in the Line* DF *produced,* &c. *And it is farther to be obſerved, that when the Angle* DCE *becomes large, the Line* nF *increaſes apace; wherefore thoſe Rays which fall near* T, *proceed after Refraction, as from a more* diffuſed *Space, than thoſe which fall at the ſame Diſtance from each other near the Point* D. *Upon which Account it is uſual with* Optical *Writers to ſuppoſe the Diſtance between the Points where the Rays enter the plain Surface of a refracting* Medium, *to be inconſiderable with Regard to the Diſtance of the* radiant *Point, if they diverge; or to that of their imaginary Focus, if they converge: and unleſs there be ſome particular Reaſon to the contrary, they conſider them, as entring the refracting* Medium *in a Direction as nearly perpendicular to its Surface as may be.*

*Caſe* 2. Of diverging Rays proceeding out of a denſer into a rarer *Medium.*

*Dem.* Let X be the denſer, Z the rarer *Medium*, FD and FE two diverging Rays proceeding from the Point F; and ſuppoſing the Perpendicular PK drawn as before, FP will be the Sine of the Angle of Incidence of the oblique Ray FE, which in this Caſe being refracted from the Perpendicular, will paſs on in ſome Line as ER, which being produced back to the Circumference of the Circle will cut the Ray FD ſomewhere, ſuppoſe in C, this there-

For being all refracted towards their reſpective Perpendiculars, they themſelves are brought nearer to a Paraleliſm, and ſo converge leſs.

V. On

fore will be the imaginary Focus of the refracted Ray ER; draw RS the Sine of the Angle of Refraction, to which HI will be equal: but here alſo FP or its equal CM, is to HI, as EC to EH, or (if the Point D and E be conſidered as contiguous) as DC to DF; that is, the Sine of the Angle of Incidence is to the Sine of the Angle of Refraction, as the focal Diſtance to that of the radiant Point. *Q. E. D.*

*Caſe* 3. Of converging Rays paſſing out of a denſer *Medium* into a rarer.

*Dem.* Let Z be the denſer, X the rarer *Medium*, and GE the incident Ray, this will be refracted from the Perpendicular into a Line as EH; then all Things remaining as before, GK, or its equal FP, or CM will be the Sine of the Angle of Incidence, and HI that of Refraction: but theſe Lines, as before, are to each other, as DC to DF, that is, the focal Diſtance is to the Diſtance of the imaginary Focus, as the Sine of the Angle of Incidence to that of the Angle of Refraction. *Q. E. D.*

*Caſe* 4. Of converging Rays paſſing out of a rarer into a denſer *Medium*.

*Dem.* Let Z be the rarer, X the denſer *Medium*, and RE the incident Ray; this will be refracted towards the Perpendicular into a Line, as EF; C will be the imaginary Focus, and F the real one, HI which is equal to RS, the Sine of the Angle of Incidence, and FP that of the Angle of Refraction: but theſe are of to each other, as DF to DC; and therefore the focal Diſtance is to that of the imaginary Focus, as the Sine of the Angle of Incidence is to that of the Angle of Refraction. *Q. E. D.*

II. When parallel Rays fall upon a ſpherical Surface of different Denſity, the focal Diſtance will be to the Diſtance of the Center of Convexity, as the Sine of the Angle of Incidence is to the Difference between that Sine and the Sine of the Angle of Refraction.

This Propoſition admits of four Caſes.

*Caſe* 1.

V. On the contrary, when they proceed out of a denſer into a rarer *Medium*, they are refracted the contrary Way, and ſo made to converge more.

All

*Caſe* 1. Of parallel Rays paſſing out of a rarer into a denſer *Medium* through a convex Surface.

*Dem.* Let AB (*Fig.* 11.) repreſent a convex Surface, C its Center of Convexity; HA and DB two parallel Rays, paſſing out of the rarer *Medium* X into the denſer Z, the one perpendicular to the refracting Surface, the other oblique: draw CB, this being a Radius, will be perpendicular to the Surface at the Point B; and the oblique Ray DB being in this Caſe refracted towards the Perpendicular, will proceed in ſome Line, as BF, meeting the other Ray in F, which will therefore be the focal Point: produce CB to N, then will DBN, or its equal BCA be the Angle of Incidence, and FBC that of Refraction. Now, *whereas any Angle has the ſame Sine with its Complement to two right ones*, the Angle FCB *being the Complement of* ACB, which is equal to the Angle of Incidence, may here be taken for that Angle; and therefore, *as the Sides of a Triangle have the ſame Relation to each other, that the Sines of their oppoſite Angles have*, FB *being oppoſite to this Angle*, and FC *being oppoſite to the Angle of Refraction*, they may here be conſidered as the Sines of the Angles of Incidence and of Refraction; and for the ſame Reaſon CB may be conſidered as the Sine of the Angle CFB, which Angle *being together with the Angle* FBC, *equal to the external one* ACB (32. *El.* 1.) is itſelf equal to the Difference between thoſe two laſt Angles; and therefore the Line FB is to CB as the Sine of the Angle of Incidence is to the Sine of an Angle which is equal to the Difference between the Angle of Incidence and of Refraction. Now, becauſe in very ſmall Angles as theſe are, *for we ſuppoſe in this Caſe alſo the Diſtance* AB *to vaniſh, the Reaſon of which will be ſhewn by and by*, their Sines bear nearly the ſame Proportion to each other that they themſelves do, the Diſtance FB will be to CB as the Sine of the Angle of Incidence is to the Difference between that Sine and the Sine of the Angle of Refraction; but *becauſe* BA *vaniſhes*, FB and FA are equal, and therefore FA is to CA in that Proportion. *Q. E. D.*

Obſ. 2.

All which may be illuſtrated in the following Manner. 1. Let AB, CD, (*Fig.* 5.) be two parallel Rays falling on the plain Surface

EF

Obſ. 2 *It appears from the foregoing Demonſtration, that the focal Diſtance of the oblique Ray* DB, *is ſuch, that the Line* BF *ſhall be to the Line* CB *or* CA *as the Sine of the Angle of Incidence to the Sine of an Angle, which Angle is equal to the Difference between the Angle of Incidence and Refraction; therefore ſo long as the Angles* BCA, &c. *are ſmall, ſo long the Line* CB *is pretty much of the ſame Length, becauſe ſmall Angles have nearly the ſame Relation to each other that their Sines have. But when the Point* B *is removed far from* A, *ſo that the Ray* DB *enters the Surface ſuppoſe about* O, *the Angles* BCA, &c. *becoming large, the Sine of the Angle of Incidence begins to bear a conſiderably leſs Proportion to the Sine of an Angle which is equal to the Difference between the Angle of Incidence and Refraction than before, and therefore the Line* BF *begins to bear a much leſs Proportion to* BC; *wherefore its Length decreaſes apace: Upon which Account thoſe Rays which enter the Surface about* O, *not only meet nearer the Center of Convexity than thoſe which enter at* A, *but are collected into a more* diffuſed *Space. From hence it is, that the Point where thoſe only which enter near* A, *are collected, is reckoned the true* Focus; *and the Diſtance* AB *in all Demonſtrations relating to the* Foci *of parallel Rays entring a ſpherical Surface whether convex or concave, is ſuppoſed to* vaniſh.

*Caſe* 2. Of parallel Rays paſſing out of a denſer into a rarer *Medium* through a convex Surface.

*Dem.* Let X be the denſer, Z the rarer *Medium*, AB the Surface by which they are ſeparated, C the Center of Convexity, and HA and DB two parallel Rays, as before. Through B the Point where the oblique Ray DB enters the rarer *Medium* draw the Perpendicular CN; and let the Ray DB, being in this Caſe refracted from the Perpendicular, proceed in the Direction BM; produce BM back to H; this will be the imaginary Focus, and DBN, or its equal ACB will be the Angle of Incidence, and CBM, or its equal HBN (for they are vertical) that of Refraction; produce DB to L and draw BF ſuch, that the Angle LBF may be equal to DBH: then becauſe NBD and DBH together are equal to NBH the Angle of Refraction, therefore BCA which is equal to the firſt, and LBF which is equal to the ſecond, are together equal to the

Angle

EF of a *Medium* of different Denſity: now becauſe they both make equal Angles of Incidence with their reſpective Perpendiculars GH, IK

Angle of Refraction; but LBF is equal to BFA (as being alternate to it) conſequently BFA and BCA together are equal to the Angle of Refraction; and therefore ſince one of them, *viz.* BCA is equal to the Angle of Incidence, the other is the Difference between that Angle, and the Angle of Refraction. Now FB the Sine of the Angle FCB, or which is the ſame Thing, of its Complement to two right ones BCA, the Angle of Incidence is to CB the Sine of the Angle BFC, as FB to CB, that is as HB to CB; for the Angles DBH and LBF being equal, the Lines BF and BH are ſo too; but the Diſtance BA vaniſhing, HB is to CB, as HA to CA: that is, the Sine of the Angle of Incidence is to the Sine of an Angle which is the Difference between the Angle of Incidence and Refraction, or becauſe the Angles are ſmall, to the Difference between the Sine of the Angle of Incidence and that of Refraction, as the Diſtance of the Focus from the Surface is to that of the Center from the ſame. *Q. E. D.*

*Caſe* 3. Of parallel Rays paſſing out of a rarer into a denſer *Medium* through a concave Surface.

*Dem.* Let X be the denſer *Medium* having the concave Surface AB, and let LB and FA be the incident Rays. Now whereas, when DB was the incident Ray, and paſſed out of a rarer into a denſer *Medium*, as in Caſe the firſt, it was refracted into the Line BF, this Ray LB having the ſame Inclination to the Perpendicular, will alſo ſuffer the ſame Degree of Refraction, and will therefore paſs on afterwards in the Line FB produced, *v. g.* towards P. So that, whereas in that Caſe the Point F was the real Focus of the incident Ray DB, the ſame Point will in this be the imaginary Focus of the incident Ray LB: But it was there demonſtrated, that the Diſtance FA is to CA, as the Sine of the Angle of Incidence is to the Difference between that and the Sine of the Angle of Refraction, therefore the focal Diſtance of the refracted Ray BP is to the Diſtance of the Center of Convexity in that Proportion. *Q. E. D.*

*Caſe* 4. Of parallel Rays paſſing out of a denſer into a rarer *Medium* through a concave Surface.

*Dem.* Let X be the rarer *Medium*, having the concave Surface AB, and let LB and FA be the incident Rays, as before. Now whereas

IK, before Refraction, they will make equal Angles of Refraction with them afterwards, and ſo proceed on in the parallel Lines BL DM.

whereas, when DB was the incident Ray paſſing out of a denſer into a rarer *Medium*, it was refracted into BM, as in Caſe the ſecond, having a Point as H in the Line MB produced, for its imaginary Focus; therefore LB, for the like Reaſon as was given in the laſt Caſe, will in this be refracted into BH, having the ſame Point H for its real Focus. So that here alſo the focal Diſtance will be to that of the Center of Convexity, as the Sine of the Angle of Incidence is to the Difference between that and the Sine of the Angle of Refraction. *Q. E. D.*

III. When diverging or converging Rays enter into a *Medium* of different Denſity through a ſpherical Surface, the *Ratio* compounded of that which the focal Diſtance bears to the Diſtance of the radiant Point (or of the imaginary Focus of the incident Rays, if they converge); and of that, which the Diſtance between the ſame radiant Point (or imaginary Focus) and the Center, bears to the Diſtance between the Center and the Focus, is equal to the *Ratio*, which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction.

This Propoſition admits of ſixteen Caſes.

*Caſe* 1. Of diverging Rays paſſing out of a rarer into a denſer *Medium*, through a convex Surface, with ſuch a Degree of Divergency, that they ſhall converge after Refraction.

*Dem.* Let BD (*Fig.* 12.) repreſent a ſpherical Surface, C its Center of Convexity, and let there be two diverging Rays AB and AD, proceeding from the radiant Point A, the one perpendicular to the Surface, the other oblique. Through the Center C produce the perpendicular one to F, and draw the Radius CB and produce it to K, and let BF be the refracted Ray; then will F be the focal Point: produce AB to H, and through the Point F draw the Line FG parallel to CB. AB being the incident Ray, and CK perpendicular to the Surface at the Point B, the Angle ABK, or which is equal to it, *becauſe of the parallel Lines* CB *and* FG, FGH, is the Angle of Incidence. Now *whereas the Complement of any Angle to two right ones has the ſame Sine with the Angle itſelf*, the Sine of the Angle FGB, *that being the Complement of* FGH *to two right ones, may be conſidered as the Sine of*

DM. 2. Let the diverging Rays AB, AE, AF, (*Fig. 6.*) paſs out of a rarer into a denſer *Medium*, through the plain Surface GH, and

*of the Angle of Incidence*; which Sine the Line FB, *as the Sides of a Triangle have the ſame Relation to each other, that the Sines of their oppoſite Angles have*, may be taken for. Again, the Angle FBC is the Angle of Refraction, or its equal, *becauſe alternate to it*, BFG; to which BG *being an oppoſite Side*, may be looked upon as the Sine. But FB is to BG in a *Ratio* compounded of FB to BA, and of BA to BG; *for the* Ratio *that any two Quantities bear to each other, is compounded of the* Ratio, *which the firſt bears to any other, and of the* Ratio *which that other bears to the ſecond.* Now FB is to BA, *ſuppoſing* BD *to vaniſh*, as FD to DA; and BA is to BG, *becauſe of the parallel Lines* CB *and* FG, as AC to CF. That is, the *Ratio* compounded of FD, *the focal Diſtance*, to DA, *the Diſtance of the radiant Point*, and of AC, *the Diſtance between the radiant Point and the Center*, to CF, *the Diſtance between the Center and the Focus*, is equal to that which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction. *Q. E. D.*

Obſ. 3. *Whereas the focal Diſtance of the oblique Ray* AB *is ſuch, that the compound* Ratio of FB *to* BA *and of* AC *to* CF *ſhall be the ſame, whatever be the Diſtance between* B *and* D; *it is evident, that ſince* AC *is always of the ſame Length, the more the Line* AB *lengthens, the more* FB *muſt lengthen too, or elſe* FC *muſt ſhorten; but it appears by Inſpection of the Figure, that if* BF *lengthens,* CF *will do ſo too, and in a greater Proportion with reſpect to its own Length than* BF *will, therefore the Lengthning of* BF *will conduce nothing towards preſerving the Equality of the Proportion: but as* AB *lengthens,* BF *and* CF *muſt both ſhorten, which is the only poſſible Way wherein the Proportion may be continued the ſame. And it is alſo apparent, that the farther* B *moves from* D, *towards* O, *the faſter* AB *lengthens, and therefore the farther the Rays enter from* D, *the nearer to the refracting Surface is the Place where they meet, but the Space they are collected in, is the more* diffuſed: *And therefore in this Caſe, as well as thoſe taken Notice of in the two foregoing Obſervations, different Rays, though flowing from the ſame Point, ſhall conſtitute different* Focus's; *and none are ſo* effectual *as thoſe which enter at or very near the Point* D. *And ſince the ſame is ob-*

and let the Ray AB be perpendicular to that Surface; the reſt being refracted towards their reſpective Perpendiculars EK, FM, will proceed

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*ſervable of converging as well as of diverging Rays, none except thoſe which enter very near that Point, are uſually taken into Conſideration; upon which Account it is, that the Diſtance* DB, *in determining the focal Diſtances of diverging or converging Rays entring a convex or concave Surface, is ſuppoſed to* vaniſh.

Thoſe who would ſee a Method of determining the preciſe Point, which the Ray AB, whether it be parallel, converging, or diverging to the Ray AF, converges to or diverges from after Refraction at B or any other given Point in the Surface DO, may find it in the Appendix to *Molineux*'s Optics, which for the Sake of thoſe who have not that Book, I ſhall ſubjoin at the End of this Note.

*Caſe* 2. Of converging Rays paſſing out of a rarer into a denſer *Medium* through a concave Surface with ſuch a Degree of Convergency, that they ſhall diverge after Refraction.

*Dem.* Let the incident Rays be HB and FD paſſing out of a rarer into a denſer *Medium* through the concave Surface BD, and tending towards the Point A, from whence the diverging Rays flowed in the other Caſe; then the oblique Ray HB having its Angle of Incidence HBC equal to ABK the Angle of Incidence in the former Caſe, will be refracted into the Line BL ſuch, that its refracted Angle KBL will be equal to FBC the Angle of Refraction in the former Caſe; that is, it will proceed after Refraction in the Line FB produced, having the ſame focal Diſtance FD with the diverging Rays AB, AD, in the other Caſe. But, by what has been already demonſtrated, the *Ratio* compounded of FD, *the focal Diſtance,* to DA, *in this Caſe, the Diſtance of the imaginary Focus of the incident Rays,* and of AC, *the Diſtance between the ſame imaginary Focus and the Center to* CF, *the Diſtance between the Center and the Focus,* is equal to that which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction. *Q. E. D.*

*Caſe* 3. Of diverging Rays paſſing out of a rarer into a denſer *Medium* through a convex Surface with ſuch a Degree of Divergency as to continue diverging.

*Dem.* Let AB, AD (*Fig.* 13.) be the diverging Rays, and let their Divergency be ſo great, that the refracted Ray BL ſhall alſo diverge from the other; produce LB back to F which will be the focal Point; draw the Radius CB and produce it to K; produce

BA

ceed in the Directions EN and FO, diverging in a leſs Degree from the Ray AP, than they did before Refraction. 3. Had they proceeded

BA likewiſe towards G, and draw FG parallel to BC. Then will ABK be the Angle of Incidence, whoſe Sine BF may be taken for, as being oppoſite to the Angle BGF, which is the Complement of the other to two right ones. And LBC is the Angle of Refraction, or its equal KBF, or which is equal to this, BFG, as being alternate; therefore BG the oppoſite Side to this may be taken for the Sine of the Angle of Refraction. But BF is to BG, for the like Reaſon as was given in Caſe the firſt, in a *Ratio* compounded of BF to BA, and of BA to BG. Now BF is to BA (DB vaniſhing) as DF to DA, and becauſe of the parallel Lines FG and BC, the Triangles CBA and AGF are ſimilar, therefore BA is to AG as CA to AF, conſequently BA is to BA together with AG, that is, to BG, as CA is to CA together with AF, that is, to CF. Therefore the *Ratio* compounded of DF the focal Diſtance to DA the Diſtance of the radiant Point, and of CA the Diſtance between the radiant Point and the Center, to CF the Diſtance between the Center and the Focus, is equal to that which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction. *Q. E. D.*

*Caſe* 4. Of converging Rays paſſing out of a rarer into a denſer *Medium* through a concave Surface in ſuch Manner that they ſhall continue converging.

*Dem.* Let HB and CD be the incident Rays paſſing out of the rarer into the denſer *Medium* through the concave Surface BD, and tending towards A the ſame Point from whence the diverging Rays flowed in the laſt Caſe. Then becauſe the Ray HB has the ſame Inclination to the Perpendicular CK that AB had before, it will ſuffer the ſame Degree of Refraction, and paſs on in the Line LB produced, having its Focus F at the ſame Diſtance from the refracting Surface as that of the diverging Ray AB in the other Caſe. Therefore, &c. *Q. E. D.*

*Caſe* 5. Of diverging Rays paſſing out of a denſer into a rarer *Medium* through a convex Surface.

*Dem.* Let AB, AD (*Fig.* 14.) be the incident Rays paſſing out of a denſer into a rarer *Medium* through the convex Surface BD whoſe Center of Convexity is C; and let BL be the refracted Ray, which produce back to F, and draw FG parallel to CB. Here ABK is the Angle of Incidence, to which its alternate one ceeded out of a denſer into a rarer *Medium*, they would have been refracted from their Perpendiculars EK, FM, and therefore have di-

verged

FGB being equal, FB the oppoſite Side may be conſidered as the Sine of it. The Angle of Refraction is LBC or FBK, of which BFG being the Complement to two right ones, BG the oppoſite Side may be looked upon as its Sine. But BF is to BG, in the compound *Ratio* of BF to BA and of BA to BG for the Reaſon given above. Now (BD vaniſhing) BF is to BA as DF to DA, and BA is to BG as CA to CF. That is, the *Ratio* compounded of the focal Diſtance to the Diſtance of the radiant Point, &c. *Q. E. D.*

*Caſe* 6. Of converging Rays paſſing out of a denſer into a rarer *Medium* through a concave Surface.

*Dem.* Let HB, CD be the incident Rays tending towards the Point A which was the Radiant in the laſt Caſe. Then, for the Reaſons already given, the oblique Ray will ſuffer ſuch a Degree of Refraction, as to have its Focus F at the ſame Diſtance from the Surface, as the diverging Rays AB, AD had in that Caſe. Therefore, &c. *Q. E. D.*

When the *Mediums* through which Rays paſs, and the refracting Surfaces are ſuch, that Rays flowing from A (*Fig.* 12.) are collected in F, then Rays flowing from F through the ſame *Mediums* the contrary Way, will be collected in A. For when Rays paſs out of one *Medium* into another, the Sine of the Angle of Incidence bears the ſame Proportion to the Sine of the Angle of Refraction, as the Sine of the Angle of Refraction does to the Sine of the Angle of Incidence, when they paſs the contrary Way. This is applicable to each of the ſix following Caſes compared reſpectively with the ſix foregoing; therefore they may be conſidered as the Converſe of them; or they may be demonſtrated independently of them, as follows.

*Caſe.* 7. Of diverging Rays paſſing out of a denſer into a rarer *Medium* through a concave Surface, ſo as to converge afterwards.

*Dem.* Let AB, AD (*Fig.* 15.) be two diverging Rays paſſing through the concave Surface BD into a rarer *Medium.* Let C be the Center of Convexity, and BF the refracted Ray. Draw CB and produce it to K, and draw FG parallel to it meeting AB produced in G. Then will ABC be the Angle of Incidence, of which FB being oppoſite to its alternate and equal Angle FGB, may be conſidered as the Sine. The Angle of Refraction is FBK, of which

GB

verged more than before. 4. Let the converging Rays AB, CD, EF (*Fig.* 7.) paſs out of a rarer into a denſer *Medium*, through the

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GB being oppoſite to its Complement to two right ones GFB, may be taken for the Sine. Now FB is to BG, in a *Ratio* compounded of FB to BA, and of BA to BG. But (BD vaniſhing) FB is to BA as FD to DA, and becauſe of the parallel Lines CB and FG, BA is to BG as CA to CF. Therefore the focal Diſtance, *&c. Q. E. D.*

*Caſe* 8. Of converging Rays paſſing out of a denſer into a rarer *Medium* through a convex Surface, ſo as to diverge afterwards.

*Dem.* Let GB and FD be the incident Rays tending towards A, and produce FB to L. Then as AB in the laſt Caſe was refracted into BF, GB will in this be refracted into BL, for the Reaſons already given, having F for its focal Point. Therefore, *&c. Q. E. D.*

*Caſe* 9. Of diverging Rays paſſing out of a denſer into a rarer *Medium* through a concave Surface, in ſuch Manner as to continue diverging.

*Dem.* Let AB, AD (*Fig.* 16.) be two Rays paſſing out of a denſer into a rarer *Medium*, through the concave Surface DB whoſe Center of Convexity is C. Draw CB, produce it to K, and let BL be the refracted Ray, produce BL back to F, and draw FG parallel to CB meeting BA produced in G. Then will ABC be the Angle of Incidence, of which FB being oppoſite to its alternate and equal Angle FGB, may be conſidered as the Sine. The refracted Angle is LBK, or its equal CBF, of which BG being oppoſite to its Complement to two right ones BFG, is the Sine. Now BF is to BG in the compound *Ratio* of BF to BA and of BA to BG: but BF is to BA as DF to DA; and becauſe of the parallel Lines CB and FG, the Triangles BCA, AGF are ſimilar, therefore BA is to AG as CA to AF, and conſequently BA is to BG as CA to CF. Therefore, *&c. Q. E. D.*

*Caſe* 10. Of converging Rays paſſing out of a denſer into a rarer *Medium* through a convex Surface, in ſuch Manner as to continue converging.

*Dem.* Let HB, MD be the incident Rays tending towards the Point A. Then will the oblique Ray HB for the Reaſons already given be refracted into BF. Therefore, *&c. Q. E. D.*

*Caſe* 11. Of diverging Rays paſſing out of a rarer into a denſer *Medium* through a concave Surface.

*Dem.*

the plain Surface GH, and let the Ray AB be perpendicular to that Surface, then the other Rays being refracted towards their reſpective

*Dem.* Let AB, AD (*Fig.* 17.) be the incident Rays paſſing out of a rarer into a denſer *Medium*, through the concave Surface BD whoſe Center of Convexity is C, and ſuppoſing the Line CB drawn and produced to K, the refracted Ray BL drawn and produced back to F, and alſo FG drawn parallel to CB, ABC will be the Angle of Incidence, of which FB being oppoſite to its Complement to two right ones BGF, is the Sine. The Angle of Refraction will be LBK or its equal FBC, of which BG being oppoſite to its equal and alternate one BFG, is the Sine. Now FB is to BG in the compound *Ratio* of FB to BA and of BA to BG. But (BD vaniſhing) FB is to BA as FD to DA, and becauſe of the parallel Lines FG and CB, BA is to BG as CA to CF. Therefore, *&c. Q. E. D.*

*Caſe* 12. Of converging Rays paſſing out of a rarer into a denſer *Medium* through a convex Surface.

*Dem.* Let HB, MD be the incident Rays tending towards A the radiant Point in the laſt Caſe; then, as was explained above, BF will be the refracted Ray. Therefore, *&c. Q. E. D.*

*Caſe* 13. Of Rays paſſing out of a denſer into a rarer *Medium* from a Point between the Center of Convexity and the Surface.

*Dem.* Let AB, AD (*Fig.* 18.) be two Rays paſſing out of a denſer into a rarer *Medium* from the Point A, which let be poſited between C the Center of Convexity and the refracting Surface BD; through B draw CK, and let BL be the refracted Ray; produce BL back to F and draw FG parallel to BC. Then will ABC be the Angle of Incidence, of which BF being oppoſite to its Complement to two right ones BGF, is the Sine. LBK will be the Angle of Refraction, or its equal FBC, of which BG being oppoſite to its alternate and equal one BFG, is the Sine. But, as before, BF is to BG in a compound *Ratio* of BF to BA and of BA to BG; and (BD vaniſhing) BF is to BA as DF to DA, and becauſe the Lines CB and FG are parallel, BA is to BG as CA to CF. Therefore, *&c. Q. E. D.*

*Caſe* 14. Of Rays paſſing out of a denſer into a rarer *Medium* towards a Point between the Center of Convexity and the Surface.

*Dem.* Let the incident Rays be MD, HB tending towards A from whence the other proceeded in the laſt Caſe. Then as in that reſpective Perpendiculars DK, FM, will proceed in the Directions DN, FN converging in a leſs Degree towards the Ray AN, than they did

that Caſe the refracted Ray BL being produced back paſſed through F, in this the refracted Ray itſelf, for the like Reaſons as were given in the foregoing Caſes, will paſs through that Point. Therefore, &c. *Q. E. D.*

*Caſe* 15. Of Rays paſſing out of a rarer into a denſer *Medium* from a Point between the Center of Convexity and the Surface.

*Dem.* Let AB, AD (*Fig.* 19.) be two diverging Rays paſſing out of a rarer into a denſer *Medium* through the refracting Surface BD, whoſe Center of Convexity is C, a Point beyond that from whence the Rays flow. Through B draw CK, and let BL be the refracted Ray, produce it back to F, and draw FG parallel to BC meeting BA produced in G. ABC will be the Angle of Incidence, of which BF being oppoſite to its alternate and equal Angle BGF, is the Sine. The Angle of Refraction is LBK or its equal FBC of which BG being oppoſite to its Complement to two right ones BFG, is the Sine. But BF is to BG in the compound *Ratio* of BF to BA and of BA to BG; and (BD vaniſhing) BF is to BA as DF to DA, and becauſe of the parallel Lines CB and GF, the Triangles AFG and ABC are ſimilar, BA therefore is to AG as CA to AF, conſequently BA is to BA and AG together, that is, to BG, as CA is to CA and AF together, that is, to CF; and therefore the focal Diſtance, &c. *Q. E. D.*

*Caſe* 16. Of Rays paſſing out of a rarer into a denſer *Medium* towards a Point between the Center of Convexity and the Surface.

*Dem.* Let HB, MD be the incident Rays having for their imaginary Focus the Point A which was the Radiant in the laſt Caſe, and let C the Center of Convexity of the refracting Surface be poſited beyond this Point. Then will HB, for the Reaſons already given, be refracted into BF, having the Point F for its real Focus which was the imaginary one of the diverging Rays AB, AD in the former Caſe. Therefore the *Ratio* compounded of that which the focal Diſtance bears to the Diſtance of the imaginary Focus of the incident Rays, and of that which the Diſtance between the ſame imaginary Focus and the Center, bears to the Diſtance between the Center and the Focus, is equal to the *Ratio* which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction. *Q. E. D.*

The

did before. 5. Laſtly, had the firſt *Medium* been the denſer, they would have been refracted the other Way, and therefore have converged more.

VI. When

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The firſt Term in the foregoing Proportion (*viz.* that in Propoſition the 3d of this Note) being always an unknown Quantity, thoſe who are not well verſed in the Uſe of ſuch Propoſitions, may think it impoſſible to inveſtigate the focal Diſtance of any refracting Surface by it: I ſhall therefore exemplify it in the following Inſtance, by which the Manner of doing it in all others will clearly be underſtood. *v. g.* Let it be required to determine the focal Diſtance of diverging Rays paſſing out of Air into Glaſs through a convex Surface, and let the Diſtance of the radiant Point be 20, and the *Radius* of Convexity be 5: Now becauſe we muſt make Uſe of the focal Diſtance before we know it, let that be expreſſed by ſome Symbol or Character as $x$: Then, becauſe by the aforeſaid Propoſition the *Ratio* compounded of that which the focal Diſtance bears to the Diſtance of the radiant Point (that is in this Suppoſition, of $x$ to 20); and of the *Ratio* which the Diſtance of the ſame radiant Point from the Center bears to the Diſtance between the Center and the Focus (in this Caſe, of 25 to $x—5$) is equal to the *Ratio* which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction (that is, of 17 to 11), we ſhall have in the Inſtance before us, the following Proportion, *viz.* $\left.\begin{array}{l} x : 20 \\ 25 : x—5 \end{array}\right\} :: 17 : 11$, and compounding them into one, which is done by multiplying the two firſt Parts together, we have $25x : 20x—100 :: 17 : 11$, and multiplying the extream Terms and middle Terms together, $340x—1700=275x$, which Equation after due Reduction gives $x=\frac{1700}{65}$.

In ſome Caſes which might have been put, the Quantity 65 would have been negative, and then the Quotient ariſing from 1700 divided by that, would have been ſo too; that is $x$ the focal Diſtance would have been negative, in which Caſe the *Focus* muſt have been taken on the contrary Side the Surface to that on which it was ſuppoſed to fall in ſtating the Problem; that is, it muſt have been taken on the ſame Side with the radiant Point, for in calling the Diſtance between the Center and the *Focus* $x—5$ it was ſuppoſed the *Focus* would fall on the ſame Side with the Center, or on that which is oppoſite to the radiant Point, becauſe otherwiſe that

VI. When Rays proceed out of a rarer into a denſer *Medium*, through a convex Surface, if they are parallel before Refraction, they become converging afterwards.

For

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that Diſtance muſt have been expreſſed by $x+5$, as any one may ſee by Inſpection of the 13th or 14th Figure, in which the *Focus* of diverging Rays entring a convex Surface, is ſuppoſed to fall on the ſame Side with the radiant Point.

In like Manner as this Problem was performed, a general Theorem may be raiſed to ſolve it in all Caſes whatſoever, by uſing Characters inſtead of Figures; as every one who is not unacquainted with algebraic Operations very well knows.

See this done, and applied to the Paſſage of Rays through the Surfaces of Lenſes in the ſecond Note to the following Chapter.

A Method of determining the Point which a Ray, entring a ſpherical Surface at any given Diſtance from the Vertex of it, converges to or diverges from after Refraction at the ſame. *From the Appendix to* Molineux's *Dioptrics.*

" *Prop.* To find the *Focus* of any Parcel of Rays *diverging from,* " or *converging to* a given Point in the *Axis* of a *ſpherical Lens* " [Surface] and inclined thereto under the ſame Angle; the " *Ratio* of the Sines in Refraction being known.

" Let GL (*Fig.* 20.) be the *Lens*, P any Point in its Surface, " V the Pole [Vertex] thereof, C the Center of the Sphere " whereof it is a Segment, O the Object or Point in the Axis to " or from which the Rays do proceed, OP a given Ray; and let " the *Ratio* of Refraction be as $r$ to $s$; make CR to CO as $s$ to $r$ " for the Immerſion of a Ray, or as $r$ to $s$ for the Emerſion, (that " is, as the Sines of the Angles in the *Medium* which the Ray enters, " to their correſponding Sines in the *Medium* out of which " it comes) and laying CR from C towards O, the Point R ſhall " be the ſame for all the Rays of the Point O. Then draw the " Radius PC if Need be continued, and with the Center R and " Diſtance OP ſweep a Touch of an Arch interſecting PC in Q; " the Line QR being drawn ſhall be parallel to the refracted Ray, " and PF being made parallel thereto, ſhall interſect the Axis in " the Point F, which is the *Focus* ſought. Or make it as CQ : " CP : : CR : CF, and CF ſhall be the Diſtance of the *Focus* " from the Center of the Sphere.

For in this Caſe, the Perpendiculars at the Points where the Rays enter the Surface, are all drawn from the Center of Convexity on the

“ *Dem.* Pet fall the Perpendiculars PX on the Axis, CY on the given Ray, and CZ on the refracted Ray. By the Conſtruction PF and QR are parallel, whence the Triangles QRC and PFC are ſimilar, and CR to QR, as CF to PF, that is CR to OP as CF to PF. Now CF : PF : : CZ : PX *ob ſimilia Triang.* whence CR : OP : : CZ : PX, and CR : CZ : : OP : PX. Again, CR is to CO as the Sines of Refraction by Conſtruction, that is, as *s* to *r*, or *r* to *s*; and as CR to CZ, ſo (CO=) $\frac{r}{s}$ or $\frac{s}{r}$ CR to $\frac{r}{s}$ or $\frac{s}{r}$ CZ, and ſo is PO to PX: But as PO to PX, ſo CO to CY. *Ergo* CY=$\frac{r}{s}$ or $\frac{s}{r}$ CZ, that is, CY to CZ is as the Sines of Refraction, but CY is the Sine of the Angle of Incidence, and CZ of the refracted Angle. *Ergo conſtat Propoſitio.* ———

“ Hitherto we have conſidered only *oblique Rays*; it now remains to add ſomething concerning *Rays parallel* to the *Axis*: In this Caſe the Point O muſt be conſidered as infinitely diſtant, and conſequently OP, OC, and CR are all infinite; and OP and OC are in this Caſe to be accounted as always equal, (ſince they differ but by a Part of the *Radius* of the Sphere GPVL, which is no Part of either of them) wherefore the *Ratio* of CR to OP will be always the ſame, *viz.* as *s* to *r* for immerging Rays, and as *r* to *s* for thoſe that emerge. And by this Propoſition CF is to PF in the ſame *Ratio.* It remains therefore to ſhew on the Baſe CP to find all the Triangles CPF, wherein CF is to PF in the *Ratio* given by the Degree of Refraction. This Problem has been very fully conſidered by the celebrated Dr. *Wallis* in his late Treatiſe of Algebra, p. 258, to which I refer; but I muſt here repeat the Conſtruction thereof. See *Fig.* 21 and 22.

“ Let GPVL be a *Lens*, VC or PC the *Radius* of its Sphere, and let it be required to find all the Points *f*, *f*, ſuch, as C*f* may be to P*f* in the given *Ratio* of *s* to *r* for immerging Rays, or as *r* to *s* for the emerging. Divide CV in K, and continue CV to F, that CK may be to VK, and CF to VF in the propoſed *Ratio.* Then divide KF equally in the Point *a*, and with that

“ Center

the other Side; and therefore, as the Rays are refracted towards thoſe Perpendiculars, they are neceſſarily refracted towards each other, and thereby made to converge.

*Let it be remembered, that in the foregoing Caſe and in all others where Rays are ſaid to paſs through a convex or concave Surface, the Surface is ſuppoſed to belong to that* Medium, *into which the Rays enter, not to that out of which they paſs. Thus, when Rays paſs out of a denſer* Medium *that has a convex Surface into a* Vacuum, *they are ſaid to paſs through a concave Surface, the Surface of the* Vacuum *into which they enter in that Caſe, being ſuch.*

VII. If they enter diverging, then for the ſame Reaſon, they are made to diverge leſs, to be parallel, or to converge, according to

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" Center ſweep the Circle FKF; this Circle being drawn gives " readily all the *Foci* of the parallel Rays OP, OP. for having " continued CP till it interſect the Circle in F, PF ſhall be always " equal to V*f* the Diſtance of the *Focus* of each reſpective Parcel " of Rays OP from the Vertex or Pole of the *Lens.*

" To demonſtrate this, draw the prickt Line VF, and by what " is delivered by Dr. *Wallis* in the above-cited Place, VF and CF " will be always in the ſame propoſed *Ratio.* Again, V*f* being " made equal to PF, CF and C*f* will be likewiſe equal, as are " CP, VC; and the Angles PC*f*, VCF being *ad verticem* are alſo " equal: Wherefore P*f* will be equal to VF, and conſequently " C*f* to P*f* in the ſame *Ratio* as CF to VF, whence, and by what " foregoes, the Points *f*, *f*, are the ſeveral reſpective *Foci* of the " ſeveral Parcels of Rays, OP, OP. *Q. E. D.*———

If any one would ſee how this is to be applied to all other Inſtances, he may conſult the Place.

the Degree of Divergency they have before they enter.

For if they diverge very much, their being bent towards their reſpective Perpendiculars in paſſing through the Surface, may only diminiſh their Divergency; whereas, if they diverge in a ſmall Degree it may make them parallel, or even to converge. What Degree of Divergency or Convergency before Refraction in this and the following Caſes, is neceſſary to make Rays become parallel, will be ſhewn at the End of this Chapter.

VIII. If they converge in ſuch Manner as to tend directly towards the Center of Convexity before they enter the Surface, they fall in with their reſpective Perpendiculars, and ſo paſs on to the Center without ſuffering any Refraction.

IX. If they converge leſs than their Perpendiculars, that is, if they tend to a Point beyond the Center of Convexity, they are made by Refraction to converge more; and if they converge more than their Perpendiculars, that is, if they tend towards a Point between the Center and the Surface, then by being refracted towards them, they are made to converge leſs.

This and the three foregoing Propoſitions may be illuſtrated in the following Manner.

1. Let AB, CD (*Fig.* 8.) be two parallel Rays entring a denſer *Medium* through the convex Surface DB, whoſe Center of Convex-

ity is E; and let one of theſe, *viz.* AB be perpendicular to the Surface. This will paſs on through the Center without ſuffering any Refraction, but the other being oblique to the Surface, will be refracted towards the Perpendicular ED, and will therefore be made to proceed in ſome Line, as DG, converging towards the other Ray, and meeting it in G, which Point for that Reaſon is called the *Focus.* 2. Had the Ray CD diverged from the other, ſuppoſe in the Line AD, it would, by being refracted towards its Perpendicular ED, have been made either to diverge leſs, be parallel, or to converge. 3. Let the Line ED be produced to F, and if the Ray had converged, ſo as to have deſcribed the Line FD, it would then have been coincident with its Perpendicular, and have ſuffered no Refraction at all. 4. If it had proceeded from any Point between C and F, as from H, or which is the ſame Thing, towards any Point beyond E in the Line BE produced, it would have been made to converge more, by being refracted towards the Perpendicular DE, which converges more than it; and had it proceeded from ſome Point, as I, on the other Side F, that is, towards any Point between B and E, it would then have converged more than its Perpendicular, and ſo, being refracted towards it, would have been made to have converged leſs.

X. When Rays proceed out of a denſer into a rarer *Medium*, through a convex Surface, the contrary happens in each Caſe.

For being now refracted from their reſpective Perpendiculars, as they were before towards them, if they are parallel before Refraction, they diverge afterwards; if they diverge, their Divergency is increaſed; if they converge in the Direction of their Perpendiculars, they ſuffer no Refraction; if they converge leſs than their reſpective Perpendiculars, they are made to converge ſtill leſs, to be parallel, or to diverge; if they converge more, their Convergency is increaſed. All which may clearly be ſeen by the Figure, without any farther Illuſtration, imagining the Rays AD, CD, *&c.* bent the contrary Way in their Refractions to what they were in the former Caſes.

XI. When Rays proceed out of a rarer into a denſer *Medium*, through a concave Surface, if they are parallel before Refraction, they are made to diverge.

For in this Caſe, the Perpendiculars at the Points where the Rays enter the Surface, being drawn from a Point on that Side of the Surface from which the Rays tend, if we conceive them to paſs through the Surface, they will be ſo many diverging Lines on the other Side, and therefore the Rays after they have paſſed through the ſame Points, muſt neceſſarily be rendered diverging in being refracted towards them.

XII. If they diverge before Refraction, then for the ſame Reaſon, they are made to diverge more,

XIII. Unleſs they proceed directly from the Center, in which Caſe they fall in with their Perpendiculars, and ſuffer no Refraction; or from ſome Point between the Center of Convexity and the Surface, for then they diverge more than their reſpective Perpendiculars, and therefore being by Refraction brought towards them, they become leſs diverging.

XIV. If they converge, then being refracted towards their Perpendiculars, they are either made leſs converging, parallel, or diverging, according to the Degree they converged in before Refraction.

To illuſtrate this, and the three foregoing Caſes. 1. Let AB, CD (*Fig.* 9.) be two parallel Rays entring the concave and denſer *Medium* X, the Center of whoſe Convexity is E, and the Perpendicular to the refracting Surface at the Point D, is EF; the Ray AB if we ſuppoſe it perpendicular to the Surface at B will proceed on directly to G; but the oblique one CD being refracted towards the Perpendicular DF, will recede from the other Ray AG in ſome Line as DH. 2. If the Ray CD had proceeded from A diverging in the Direction AD it would have been bent nearer to the Perpendicular, and therefore have diverged more. 3. But if it had diverged from the Center E, it would have fallen in with the Perpendicular EF, and not have been refracted at all: and had it proceeded from I, a Point on the other Side the Center E, it would by being refracted towards the Perpendicular DF have proceeded in ſome Line nearer it than it otherwiſe would have done, and ſo would diverge leſs than before Refraction. 4. If it had converged in the Line LD, it would have been rendered leſs converging, parallel, or diverging, according to the Degree of Convergency, which it had before it entered into the refracting Surface.

XV. If the ſame Rays proceed out of a denſer into a rarer *Medium* through a concave Surface, the contrary happens in each Suppoſition: The parallel are made to converge; thoſe which diverge leſs than their reſpective Perpendiculars, that is, thoſe which proceed from a Point beyond the Center, are made leſs diverging, parallel, or converging, according to the Degree in which they diverge before Refraction; thoſe which diverge more than their reſpective Perpendiculars, that is, thoſe which proceed from a Point between the Center and the refracting Surface, are made to diverge ſtill more. And thoſe which converge, are made to converge more. All which may eaſily be ſeen by conſidering the Situation of the Rays AD CD, *&c.* with Reſpect to the Perpendicular EF; and therefore requires no farther Illuſtration.

XVI. When

Part III. Plate I. *Pag.* 42.

*Fig.* 1. *P.* 7.

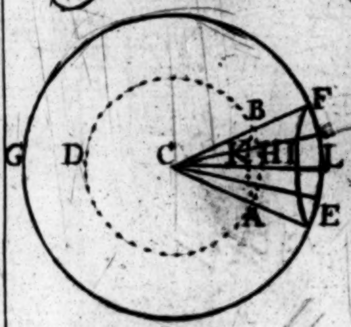


*Fig.* 2. *P.* 11.

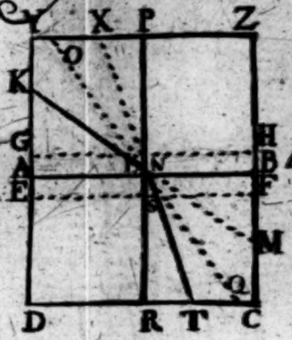


*Fig.* 3. *P.* 15.

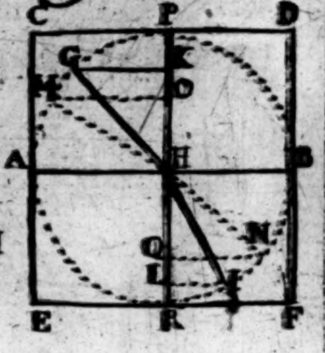


*Fig.* 4. *P.* 15.

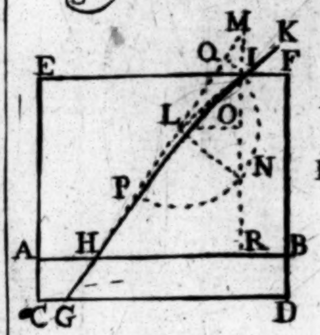


*Fig.* 5. *P.* 24.

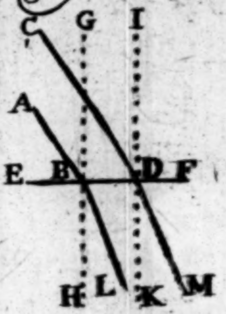


*Fig.* 6. *P.* 27.

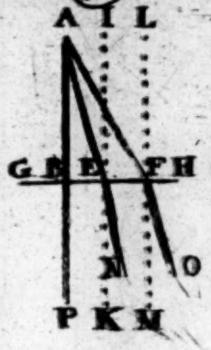


*Fig.* 7. *P.* 3

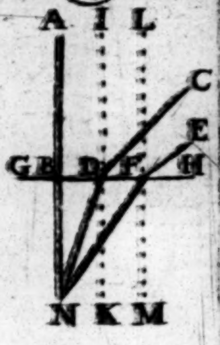


*Fig.* 8. *P.* 38.

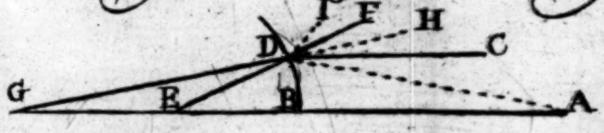


*Fig.* 10. *P.* 20.

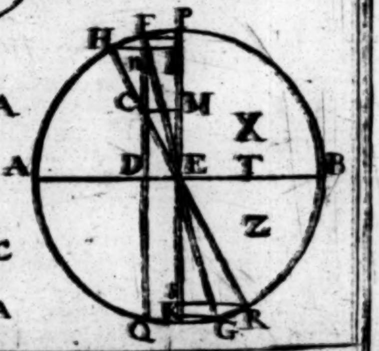
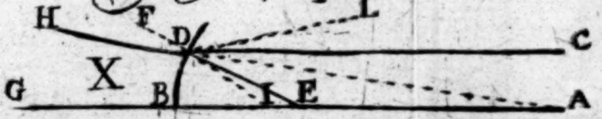


*Fig.* 9. *P.* 41.



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**Part III. Plate II.** *Pag. 42*

*Fig.* 11. *P.* 23.

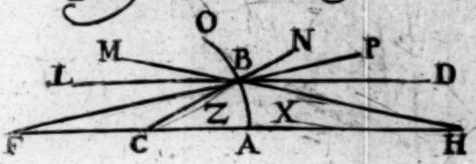


*Fig.* 12. *P.* 26.

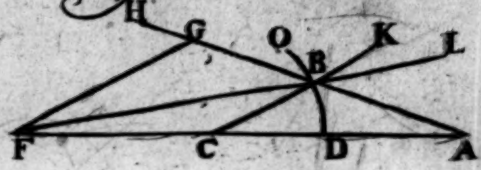


*Fig.* 13. *P.* 28.

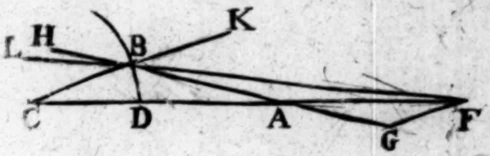


*Fig.* 14. *P.* 29.

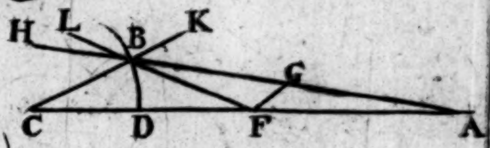


*Fig.* 15. *P.* 30.

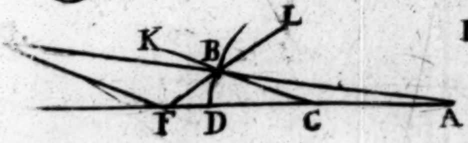


*Fig.* 16. *P.* 31.

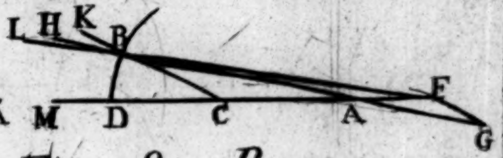


*Fig.* 17. *P.* 32.

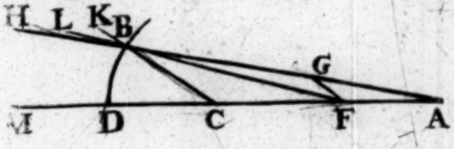


*Fig.* 18. *P.* 32.

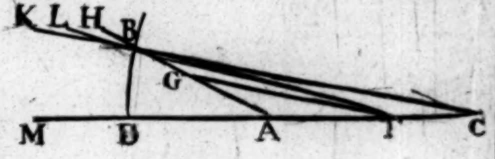


*Fig.* 19. *P.* 33.

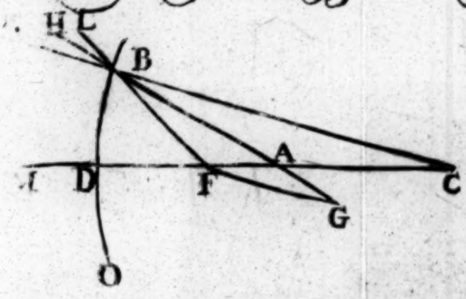
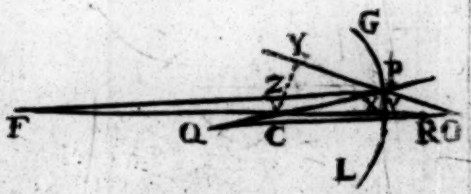


*Fig.* 20. *P.* 35.



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XVI. When diverging Rays are by Refraction made to converge, the nearer their radiant Point is to the refracting Surface, the farther is their *Focus* from it on the other Side, and *vice verſâ.*

For the nearer the radiant Point is to the refracting Surface, the more the Rays which fall upon the ſame Points of it, diverge before Refraction, upon which Account they converge the leſs afterwards.

XVII. When the radiant Point is at that Diſtance from the Surface, at which parallel Rays coming through it from the other Side would by Refraction be collected, then Rays flowing from that Point become parallel on the other Side, and are ſaid to have their *Focus* at an infinite Diſtance. For the Power of Refraction in the *Medium* is the ſame, whether the Ray paſſes one Way or the other. For Inſtance, if the parallel Rays AB, CD (*Fig.* 8.) in paſſing through the refracting Surface BD are brought to a *Focus* in G, then Rays flowing from G as a radiant Point will afterwards proceed in the parallel Lines BA and DC. And the Point G, where the parallel Rays AB and CD meet, is called the *Focus of parallel Rays.*

XVIII. When Rays proceed from a Point nearer the refracting Surface than the *Focus* of parallel Rays, they continue to diverge after Refraction, and their *Focus* is then an imaginary

nary one, and ſituated on the ſame Side the Surface with the Radiant.

For in this Caſe, their Divergency being greater than that which they would have, if they had proceeded from the *Focus* of parallel Rays, they cannot be brought to a Paralleliſm with one another, much leſs be made to converge, and therefore they continue to diverge, though in a leſs Degree than before they paſſed through the refracting Surface; upon which Account, they proceed after Refraction, as if they came from ſome Point farther diſtant from the refracting Surface than their Radiant.

## CHAP. IV.

## *Of* Lenſes, *and the Manner in which Rays are affected in paſſing through them.*

A *Lens*, is a *Medium* terminated on one Side by a ſpherical Surface, on the other by a Surface either plain or ſpherical. And of theſe there are five Sorts. The firſt, as A, (*Fig.* 23.) is plain on one Side and convex on the other; the ſecond, B, convex on both Sides; the third, plain on one Side and concave on the other, as C; the fourth, D, concave on both Sides; the fifth, convex on one Side and concave on the other, as E, which is by ſome called a *Meniſcus*.

The *Axis* of a *Lens* is a Line paſſing perpendicularly through both its Surfaces: Thus, the Line FG is an *Axis* common to all the five.

*Lenſes* are diſtinguiſhed into two general Kinds, convex and concave; the firſt and ſecond *Lenſes* are conſidered, as convex; the third and fourth, as concave: the laſt, if its Convexity is greater than its Concavity, is looked upon, as convex; if on the contrary, it is conſidered as concave.

A *Lens* is always ſuppoſed to conſiſt of a *Medium* denſer than the circumambient one, unleſs where the contrary is expreſſed.

When parallel Rays fall upon the Surface of a convex *Lens*, they are refracted towards each other in paſſing through it, and thereby collected to a *Focus* on the other Side.

To explain this, let us trace the Progreſs of a Ray as AB (*Fig.* 24.) through the convex *Lens* CDEH, whoſe Axis is IK. Let L be the Center of the firſt Convexity CDE, and M that of the other CHE; and let the Ray AB be parallel to the Axis; through B draw the Line LN which will be perpendicular to the Surface CDE at that Point. The Ray AB in entring the denſer Subſtance of the *Lens* will be refracted towards the Perpendicular, and therefore proceed after it has entered the Surface at B in ſome Direction inclined towards the Axis as BP. Through M the Center of Convexity of this Surface and the Point P draw

the

the Line MR, which paſſing through the Center will be perpendicular to the Surface at P, and the Ray now entering a rarer *Medium* will be refracted from the Perpendicular into ſome Direction as PF. In like Manner, and for the ſame Reaſons, the parallel Ray ST on the other Side the Axis, and alſo all the intermediate ones as XZ, *&c.* will meet it in the ſame Point, unleſs the Rays AB and ST enter the Surface of the *Lens* at too great a Diſtance from the Axis IF, the Reaſon of which has already been fully explained *.

The Point F where the parallel Rays AB, SF, *&c.* are ſuppoſed to be collected by paſſing through the *Lens* CE, is called *the Focus of parallel Rays of that Lens*.

If the Rays converge before they enter the *Lens*, they are then collected at a Point nearer to the *Lens* than the *Focus* of parallel Rays. If they diverge before they enter the *Lens*, they are then collected in a Point beyond F; unleſs they proceed from a Point on the other Side at the ſame Diſtance with the *Focus* of parallel Rays, in which Caſe they are rendered parallel. If they proceed from a Point nearer than that, they diverge afterwards, but in a leſs Degree than before they enter the *Lens*.

If the *Lens* is plain on one Side and convex on the other, the Rays are refracted the ſame Way, but in a leſs Degree.

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* See *Obſervation* 3, in the foregoing Note.

Had the Rays AB, ST proceeded from a radiant Point on one Side the *Lens*, and been collected in a *Focus* on the other; then if they ſhould be ſuppoſed to proceed from that focal Point as from a Radiant, and paſs through the *Lens* the contrary Way, they would be collected in that Point which was the Radiant in the other Caſe: and the nearer the Radiant Point is to the *Lens*, the farther is the *Focus* from it on the other Side, and *vice verſâ.*

If the Rays AB, CD, EF, *&c.* (*Fig.* 25.) parallel to each other, but oblique to GH the Axis of the *Lens* IK, or if the diverging Rays CB, CF, proceed as from ſome Point C which is not ſituated in the Axis of the *Lens*, they will be collected into ſome Point as L, not directly oppoſite to the Radiant C, but nearly ſo: for the Ray CD which paſſes through M the Middle of the *Lens* and falls upon the Surface of it with ſome Obliquity, will itſelf ſuffer a Refraction at D and N; but then it will be refracted the contrary Way in one Place to what it is in the other, and theſe Refractions will be equal in Degree if the *Lens* has an equal Convexity on each Side, as we may eaſily perceive if we imagine ND to be a Ray paſſing out of the *Lens* both at N and D, for it is evident the Line ND has an equal Inclination to each Surface at both its Extremities. Upon which Account the Difference between the Situation of the Point L and one directly oppoſite to C, is ſo ſmall, that it is generally neglected; and the *Focus* is ſuppoſed to be in that Line, which a Ray, that would paſs through the middle Point of the *Lens*, were it to ſuffer no Refraction, would proceed in.

All which is ſufficiently clear, from what has been ſaid concerning the Laws of Refraction explained in the foregoing Chapter.

When parallel Rays fall upon a concave *Lens*, they are refracted from each other in paſſing through it, and thereby made to diverge, proceeding as from an imaginary *Focus* on the firſt Side the *Lens*.

In order to comprehend this, let AB CD (*Fig.* 26.) repreſent a concave *Lens*, EF its Axis, GH the Radius of the firſt Concavity, IK that of the ſecond; produce HG to L, and let MG be a Ray of Light entring the *Lens* at the Point G. This Ray being refracted towards the Perpendicular GL, will paſs on to ſome Point as K in the other Surface more diſtant from the Axis than G, and being there refracted from the Perpendicular, IK will be diverted farther ſtill from the Axis, and proceed in the Direction KN as from ſome Point as O on the firſt Side of the *Lens*. In like Manner other Rays as PQ parallel to the former, will proceed after Refraction at both Surfaces as from the ſame Point O; which upon that Account will be the imaginary *Focus* of parallel Rays of this *Lens*.

If the Rays diverge before they enter the *Lens*, their imaginary *Focus* is then nearer the *Lens* than that of the parallel Rays. If they converge before they enter the *Lens* proceeding towards ſome diſtant Point in the Axis as E, they are then rendered leſs converging: if they converge to a Point at the ſame Diſtance from the *Lens* with the *Focus* of parallel Rays, they then go out parallel; if to a Point at a leſs Diſtance they remain converging, but in a leſs Degree than before they entered the *Lens*.

When the Rays enter the *Lens* diverging, the nearer their radiant Point is to it, the nearer alſo is their imaginary *Focus* after Refraction, and *vice verſâ*.

If the *Lens* is plain on one Side and concave on the other, the Rays ſuffer a like Refraction in each Caſe, but in a leſs Degree.

The Truth of what has been ſaid concerning the Paſſage of Rays through a concave *Lens*, is eaſily to be deduced from the Laws of Refraction delivered in the foregoing Chapter.

But the Method of determining the exact focal Diſtances of *Lenſes* is to be had from the Propoſitions laid down and demonſtrated in the Note in the foregoing Chapter. Thus, the Progreſs of the Rays after their Refraction at the firſt Surface where they enter a *Lens*, is had by one of thoſe which determines the focal Diſtance of Rays entring a denſer *Medium* of ſuch Form: And their Progreſs after their Re-

Refraction at the other Surface where they go out, is had by computing what Progreſs Rays, moving in the Direction they are found to have after their Entrance at the firſt Surface, will acquire by being refracted at the other; which is to be effected by one which determines the focal Diſtance of Rays paſſing out of a denſer *Medium* of like Form with that of the *Lens* [*].

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[*] Or a general Theorem may be made after the following Manner, to determine the Progreſs of Rays after Refraction at both Sides of the *Lens*, whatever be the Matter of it, or the Form wherein it is made.

Thus ſuppoſe GH (*Fig.* 28.) to be a given *Lens*, and E a Point in its Axis from whence the diverging Rays EL, *&c.* fall upon the *Lens*, AL the *Radius* of the firſt Convexity, and CK that of the ſecond; let LK$f$ be the Direction of the diverging Ray EL after its Refraction at the firſt Surface, and KF its Direction after Refraction at both. Then will $f$ be the *Focus* of the Rays after their firſt Refraction, and F the Point they will meet in after both. Let BD be the Thickneſs of the *Lens*, and let the Proportion which the Sine of the Angle of Incidence bears to the Sine of the Angle of Refraction be expreſſed by the *Ratio* of I to R. Call EB, $d$; BD, $t$; AB, $r$; CD, $s$; B$f$, $x$; DF, $y$: Now, to find $f$ their *Focus* after Refraction at L where they enter the firſt Surface of the *Lens*, comes under the third Propoſition in the forementioned Note: According to which the *Ratio* compounded of $x$, *the focal Diſtance ſought*, to $d$, *the Diſtance of the radiant Point*; and of $d+r$, *the Diſtance between the ſame Point and the Center*, to $x-r$, *the Diſtance between the Center and the* Focus, is as I to R; compounding theſe two *Ratios* therefore (that is multiplying them together) we have $dx+rx : dx-dr :: I$ to R; which Proportion being converted into an Equation, and duly reduced, gives $x = \frac{Idr}{Id-Rd-Rr}$.

Thus having found the Diſtance B$f$, and conſequently the Point $f$, to which the Rays converge from L, we muſt proceed to find F, that to which they will converge after having paſſed through K, where

When a Ray paſſes through a *Medium* terminated by two plain and parallel Surfaces, it
is

where they ſuffer a ſecond Refraction: This comes under the ſame Propoſition, but if we would uſe the ſame Letters as before, to expreſs the Proportion which the Sine of the Angle of Incidence bears to that of the Angle of Refraction, they muſt be put one for the other; becauſe when Rays paſs out of a denſer into a rarer *Medium*, the Sine of the Angle of Incidence bears the ſame Proportion to the Sine of the Angle of Refraction, that the Sine of the Angle of Refraction does to the Sine of the Angle of Incidence, when they paſs out of of a rarer into a denſer. This being obſerved, by the aforeſaid Propoſition we ſhall have the *Ratio* compounded of $y$, *the focal Diſtance*, to $\frac{Idr}{Id-Rd-Rr}-t$, *the imaginary Focus of the incident Rays*; and of $\frac{Idr}{Id-Rd-Rr}-t+s$, *the Diſtance between the imaginary Focus and the Center*, to $y+s$, *the Diſtance between the Center and the Focus*, as R to I. Which Equation, if we reduce the mixed Quantities $\frac{Idr}{Id-Rd-Rr}-t$, and $\frac{Idr}{Id-Rd-Rr}-t+s$ into improper Fractions, will ſtand thus,

$$\left.\begin{array}{c} y : \dfrac{Idr-Idt+Rdt+Rrt}{Id-Rd-Rr} \\ \text{and} \\ \dfrac{Idr-Idt+Rdt+Rrt+Ids-Rds-Rrs}{Id-Rd-Rr} : y+s \end{array}\right\} :: I : R.$$

And compounding theſe Ratios we have

$$\frac{Idry-Idty+Rdty+Rrty+Idsy-Rdsy-Rrsy}{Id-Rd-Rr} : \frac{Idry-Idty+Rdty+Rrty+Idrs-Idts+Rdts+Rrts}{Id-Rd-Rr} :: R : I.$$

And multiplying Extreams together and Means together, we have $IIdry-IIdty+IRdty+IRrty+IIdsy-IRdsy-IRrsy=IRdry-IRdty+RRdty+RRrty+IRdrs-IRdts+RRdts+RRrts$; which Equation being reduced, gives

$$y=\frac{IRdrs-IRdts+RRdts+RRrts}{IIdr-IIdt+2IRdt+IRrt+IIds-IRds-IRrs-IRdr-RRdt-RRrt}.$$

G This

is refracted in going out of one, as much as it was in entring the other; and therefore proceeds afterwards not in the ſame Direction, but in one that is parallel to that which it had before. Thus if the Ray AB (*Fig.* 27.) enters the denſer *Medium* CDEF terminated by the parallel Surfaces CD and EF, it is refracted at B towards the Perpendicular BI, proceeding to a Point as G, where it is as much refracted from the Perpendicular GK in going out, and proceeds in the Direction GH not the ſame, but parallel to the former ABL.

This Theorem may be applied to all Caſes whatever, even to plain Surfaces *mutatis mutandis*; *v. g.* the *Radius* of a concave Surface being negative (as lying the contrary Way) with Reſpect to that of a Convex, and the *Radius* of a plain Surface being an infinite Line; if we would apply this Theorem to a concave Surface, we muſt change all the Sines of thoſe Members wherein the Symbol expreſſing the *Radius* of that Surface occurs; and if to a plain Surface, all the Members which involve the *Radius* muſt be conſidered as infinite Quantities, that is, all, except them, muſt be ſtruck out of the Equation as nothing. So likewiſe, if we would have it extend to other Rays beſides diverging ones, the Point where converging Rays would meet, lying on the contrary Side to that from whence the diverging ones were ſuppoſed to flow, its Diſtance muſt be made negative; and the Diſtance where parallel Rays meet being infinite, it is only changing the Sines of all thoſe Members in which *d* is found, if the Rays are ſuppoſed converging, or making thoſe Members infinite in Caſe the Rays are ſuppoſed parallel; which is done by ſtriking out all the reſt, as bearing no Proportion to them.

See the Method of reducing this Equation to fewer Terms, where it is alſo illuſtrated with divers Inſtances, in Dr. *Browne*'s Appendix to *Gregory*'s Optics; or in Dr. *Halley*'s Method of finding the principal Focus of Optic Glaſſes univerſally; Philoſoph. Tranſact. No. 205.

# CHAP. V.

## *Of the* Eye.

THe Form of the Eye is ſuch as is repreſented in Figure 29, and would be a perfect Sphere, were not the fore Part AA ſomewhat more protuberant than the reſt.

The Deſcription of it, ſo far as is neceſſary to explain the Nature of Viſion, is as follows.

It is incloſed in three diſtinct Coats or Teguments; the outermoſt of which, *viz. aa*, is called *Tunica Sclerotica*; the next *cc*, *Choroides*, or *Uvea*; the third and innermoſt *dd*, derives its Name from that of its Diſcoverer, and is called *Tunica Ruyſchiana.*

Theſe Coats are contiguous to each other every where, except on the fore Part of the Eye.

That Portion of the *Sclerotica* which lies between A and A, is more protuberant than the reſt, is tranſparent, and has the Name of *Tunica Cornea.*

That Portion of the *Choroides* which is ſituated between *b* and *b*, is called the *Iris*; and is that which by its Colour denominates an Eye black, grey, *&c.* In the Middle of this there is a round Hole as *pp*, called the *Pupil.*

The *Iris* conſiſts of two Kinds of muſcular Fibres; the firſt lies extended from its Extremity like the *Radii* of a Circle, and point towards the Middle of the Pupil as towards a Center : the other are circular ones and ſurround the Pupil having the Middle of it for their common Center. Theſe are connected to the former where they croſs : and therefore when theſe contract, the Pupil is diminiſhed ; when the other, it is inlarged.

Within the Cavity of the Eye, and not far behind the Pupil, there is a ſoft tranſparent Subſtance CC, not unlike a double convex *Lens*, one of whoſe Surfaces as S, is more convex than the other. This is called the *Chriſtalline Humour* ; and is ſuſpended within the Eye by certain Ligaments as C*l*, C*l*, called *Ligamenta Ciliaria*, or *Proceſſus Ciliares* : theſe are convex towards the Pupil, as expreſſed in the Figure, and concave on the other Side, and are muſcular, and therefore capable of Contraction and Dilatation. The convex Side of theſe Ligaments is lined with a very black *Mucus* or Subſtance, as is alſo that Side of the *Iris* which is next it.

The *Tunica Ruyſchiana* leaves the *Choroides* at *l*, and paſſing behind the *Ligamenta Ciliaria* and the Chriſtalline Humour, is contiguous to them, and joins the *Choroides* again at *l*, on the other Side the Pupil.

By means of the forementioned Parts, the Cavity of the Eye is divided into two Portions ; the one of which VV is filled with a Fluid nearly of the ſame Denſity with Water, and is therefore called *Humor Aqueus*; the other TT contains a Fluid whoſe Conſiſtency is greater than that of the former, and is called *Humor Vitreus.*

At the back Part of each Eye, but not directly oppoſite to the Pupil, there enters a Nerve as NN, which is called the *Optic Nerve.*

The Fibres of this Nerve, after their Entrance into the Eye at N, ſpread themſelves over the innermoſt Coat of it as far as the *Ligamenta Ciliaria*, and form a very thin Membrane, called *Tunica Retina.*

The innermoſt Coat of the Eye is every where covered (except that Part of it which is contiguous to the back Part of the Chriſtalline Humour) with a very black *Mucus*, not unlike that with which the back Part of the *Iris* and fore Part of the *Ligamenta Ciliaria* were obſerved to be cover'd. This is to hinder any Light from being reflected from thoſe Parts on to the *Retina*; which would render the Images of Objects indiſtinct, as we ſhall ſee when we have explained the Nature of Viſion, which is the Subject of the following Chapter.

## CHAP. VI.

## *Of the Nature of* Viſion.

SUch is the Subſtance and Form of the Humours of the Eye, when lodged in their proper Receptacles, that Rays of Light in paſſing through them are affected in the like Manner as in paſſing through a convex *Lens*, as we ſhall ſee immediately; and therefore to underſtand the Nature of Viſion, which depends on the Paſſage of Rays flowing from the *ſeveral* Points of a diſtant Object through thoſe Humours, little more is required than to know how the ſame Rays would be affected, were they ſuppoſed to paſs through a convex *Lens*. Which may eaſily be done by applying to this Caſe what has been delivered in the fourth Chapter concerning the Manner in which Rays flowing from a *ſingle* Point, are affected in paſſing through *Lenſes* of that Kind.

We have already ſeen, in the abovementioned Chapter, that Rays flowing from a *ſingle* radiant Point, and afterwards falling on a convex *Lens*, are collected to a *Focus* in ſome Point oppoſite, or nearly ſo, to the Radiant. Let us now ſuppoſe an Object placed before a *Lens*, but farther from it than the focal Diſtance of its parallel Rays, and let it ſend forth

Rays from each Point of its Surface in every Direction, as from ſo many radiant Points. Some of the Rays which flow from each Point of that Surface of the Object which is turned towards the *Lens* will neceſſarily fall upon it, and paſſing through it will be collected in ſo many diſtinct focal Points on the oppoſite Side, as there are diſtinct radiant Points in the Surface of the Object from whence they came. Now as the radiant Points are contiguous to each other in the Surface of the Object on one Side of the *Lens*, the focal Points will alſo be contiguous on the other; and as each focal Point is oppoſite to its reſpective Radiant, their Places will have the ſame Relation to each other, that thoſe of the Radiants have; and conſequently theſe Points taken together will be a true Repreſentation and perfect Image of that Object; for each Point will exhibit the ſame Colour that its correſpondent Point in the Object is of. But becauſe each Point in the Repreſentation is oppoſite to its reſpective one in the Object, the Image will be inverted. The Truth of this may eaſily be experienced, if we hold a clean white Paper facing the *Lens* in the Place where the focal Points are, and take Care to prevent all other Light from falling upon the Paper, except that which paſſes through the *Lens* *.

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* On this depends the Structure of the *Obſcura Camera*, which is a Contrivance to exhibit the Repreſentation of ſuch Object as may

To illuſtrate this; Let PQR (*Fig.* 30) repreſent an Object placed before the *Lens* AB, and ſending forth Rays from each Point in its Surface; and let $q$ be the focal Diſtance of Rays proceeding from Q and paſſing through the ſaid *Lens*. Then will all the Rays that proceed from the Point Q between the Lines QA and QB be collected in $q$; in like Manner all that flow from P between the Lines PA and PB, will meet in the oppoſite Point $p$; and ſo many as proceed from R and paſs thorugh the *Lens* will be collected in $r$; and all the Rays that flow from the remaining Points between P and R and fall upon the *Lens*, will be collected in as many Points between $p$ and $r$; and $pqr$, if the Rays are received there upon a white Surface, will exhibit an Image of the Object PR, but inverted; becauſe the Rays PL$p$ and RL$r$

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may be ſeen from a Window, upon ſome plain white Surface held before the Window within the Room. In order to do this, a common ſpectacle Glaſs or Burning Glaſs (both which are convex *Lenſes*) muſt be fixed in an Hole in the Windowſhut; for then if no Light be ſuffered to enter into the Room, but what paſſes through the Hole, and a Sheet of white Paper be held oppoſite to the Hole at that Diſtance where the Rays proceeding from the Objects abroad, and paſſing through the Glaſs are collected into their reſpective *Foci*; we ſhall have the Images of all the Objects which lie before the Hole, repreſented upon the Paper, inverted; but in a much more lively and exact Manner than can be done by the Pencil; and not only the Objects and their reſpective Situations, but, what is peculiar to this Sort of Painting, their Motions will alſo be expreſſed.

**Part III. Plate III.** *Pag.* 58

*Fig.* 21. *P.* 36.

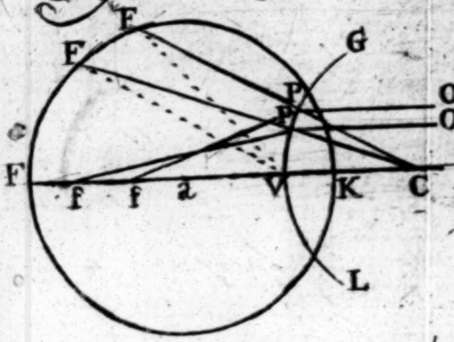


*Fig.* 22. *P.* 36.

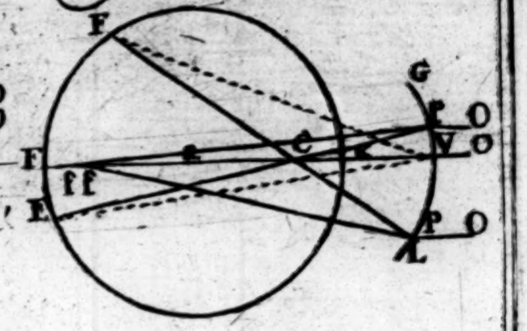


*Fig.* 23. *P.* 44.

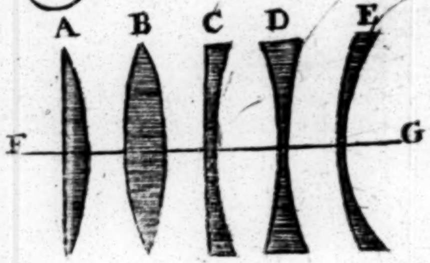


*Fig.* 24. *P.* 45.

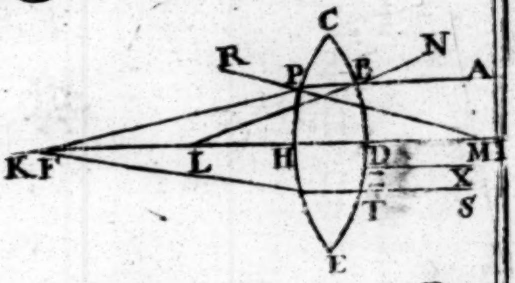


*Fig.* 25. *P.* 47.

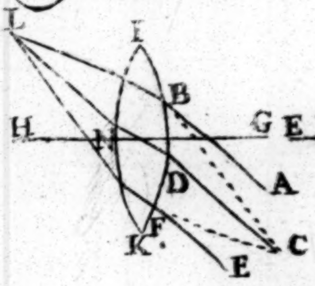


*Fig.* 26. *P.* 48.

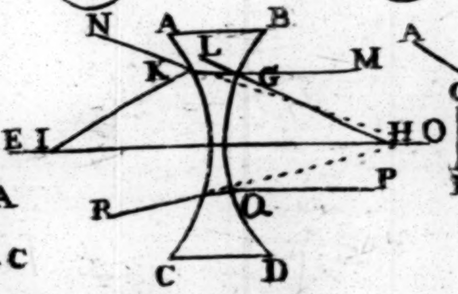


*Fig.* 27. *P.* 52.

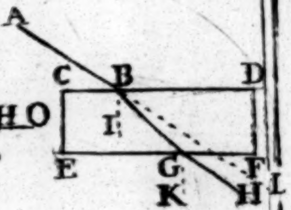
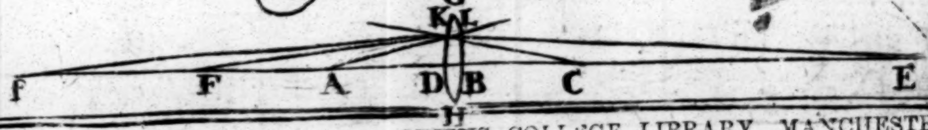


*Fig.* 28. *P.* 50.



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croſs each other at L in paſſing through the *Lens* *.

Thoſe Rays which flow from the ſame Point of an Object, when conſidered together, are called a *Cone* or *Pencil* of Rays. Thus the Rays QA, QL, *&c.* conſtitute a Pencil flowing from the Point Q; ſo the Rays PA, PL, *&c.* a Pencil from the Point P; and the middle Ray of each Pencil, as PL, QL, *&c.* is called the *Axis* of that Pencil, to which it belongs.

Now in like Manner as the ſeveral Pencils of Rays flowing from the diſtinct Points in the Surface of an Object placed before a *Lens*, are collected in ſo many Points at a certain Diſtance on the other Side of the *Lens*, and form an Image there when received upon a white Paper; ſo Pencils proceeding from an Object placed before the Eye, and being refracted in paſſing through the Humours of it, are collected into their reſpective *Foci* upon the *Retina*, where they form a Repreſentation of that Object; and by their Impulſes upon the tender Nerves of the *Retina*, an *Idea* of the Object is excited in the Mind.

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* A burning Glaſs, is no other than a Piece of Glaſs ground into the Form of a convex *Lens*; for if the Rays of the Sun are permitted to paſs through ſuch an one, they will burn very ſtrongly in the Place where they are collected into their reſpective *Foci*; upon which Account it is, that the Point where Rays in general are collected, is called their *Focus*, that is, their Place of Burning.

The Progreſs of the Rays through the Humours of the Eye, are expreſſed in the 31ſt Figure: where FG*t* is the Eye, FG the *Tunica Cornea*, *pp* the Pupil, AA the aqueous Humour, HH the chriſtalline, and VV the vitreous. And RS repreſents an Object placed before it, emitting Pencils of Rays from its ſeveral Points R, S, T, *&c.* The Rays which conſtitute the Pencil GTF, in entring the aqueous Humour, paſs out of a rarer into a denſer *Medium* through a convex Surface, in which Caſe diverging Rays are made to diverge leſs, to become parallel or to converge (Chap. 3. Prop. 7.); in entring the chriſtalline they do the like; and in paſſing out of that, they enter a rarer through a concave Surface (*viz.* the concave Surface of the vitreous) which alſo has the ſame Effect (Chap. 3. Prop. 15.) By which Means they are made to converge, as deſcribed in the Figure, and to meet together in a *Focus* at *t* a Point in the *Retina.* In like Manner the Rays flowing from R, and conſtituting the Pencil GRF, will proceed as deſcribed in the Figure, and after Refraction meet in *r*; and the Rays proceeding from S will be collected in *s*, *&c.* by which Means an Image of the Object will be formed in *rts* upon the *Retina*, but becauſe the Pencils croſs each other in paſſing through the Pupil, it will be inverted *.

The

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* Of this we have experimental Proof: For if we cut away the back Part of an Eye, and apply a Paper there, we ſhall ſee the

Image

The leſs the Diſtance between the Object and the Eye is, the more the Rays which come from the Object, are ſaid to diverge, and *è contra:* Not that the Situation of the Eye makes any Alteration in the Progreſs of thoſe Rays, but that, when the Eye is placed nearer the Object, it receives into its Pupil Rays which diverge in a greater Degree than thoſe which it can receive when placed farther off. The following Illuſtration will make this clear: Let AB (*Fig.* 32.) repreſent an Object emitting Rays from each Point of its Surface, and let C*d*, C*r*, *&c.* expreſs thoſe which flow from the Point C: let *m* be a Pupil of an Eye placed at the Diſtance C*m* from it; 'tis plain this Pupil will receive into it the diverging Rays C*r*, C*s*; whereas the Rays C*o*, C*t* will diverge the moſt of any that can enter the ſame Pupil, when placed at the Diſtance C*n*; but theſe diverge leſs than the former, the Angle *o*C*t* being included in the Angle *r*C*s*.

Images of external Objects placed thereon, as accurately as in the *Obſcura Camera*, provided no Light is permitted to fall upon the Paper, except that which paſſes through the Humours of the Eye.

Optic Writers have made it Matter of great Difficulty to determine the Point where the *Axes* of the Pencils which enter the Eye, croſs each other; ſome placing it in the Center of the Eye, others in the Vertex of the chriſtalline Humour, others in that of the *Tunica Cornea*: But as the Rays of each Pencil fill the Pupil, or as the Pupil itſelf is a common Baſe to each Pencil, it is inconceivable how the *Axes* of thoſe Pencils ſhould croſs each other in any other Place than the Center of the Pupil. See Figure 31, or any other where ſeveral Pencils are repreſented, as paſſing through the Pupil of an Eye.

Viſion is diſtinguiſhed into *bright* and *obſcure*; *diſtinct* and *confuſed*.

It is ſaid to be *bright*, when a ſufficient Number of Rays enter the Pupil at the ſame Time; *obſcure*, when too few. It is *diſtinct*, when each Pencil of Rays is collected to a *Focus* exactly upon the *Retina*; *confuſed*, when they meet before they come at it, or when they would paſs it before they meet; for in either of theſe laſt Caſes, the Rays flowing from different Points of the Object, will fall upon the ſame Part of the *Retina*, which muſt neceſſarily render the Image confuſed and indiſtinct.

Now that Objects may appear with a due *Brightneſs*, whether more or fewer Rays proceed from them, we have a Power of contracting, or dilating the Pupil by means of the muſcular Fibres of the *Iris* (as explained in the foregoing Chapter), in order to take in more or fewer Rays as Occaſion requires. But this Power has its Limits.*

And that the Rays may be collected into Points exactly upon the *Retina*, that is, that Objects may appear *diſtinct*, whether they be nearer or farther off, that is, whether the Rays proceeding from them diverge more or leſs, we have a Power of contracting or relaxing the *Ligamenta Ciliaria*, and thereby altering the

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* In ſome Animals this Power is much greater than in others; particularly in ſuch as are obliged to make Uſe of their Eyes by Night, as well as by Day, as in Cats, *&c.*

Part III. Plate IV. Pag. 62.

Fig. 29. P. 53.

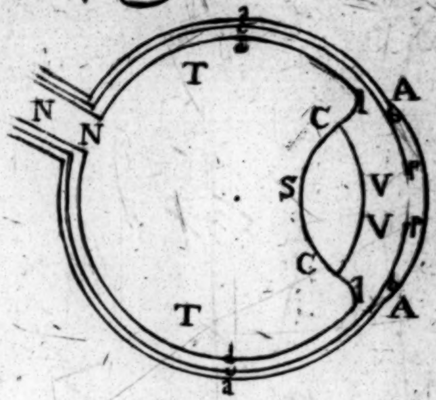


Fig. 30. P. 58.

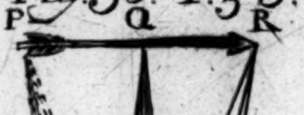
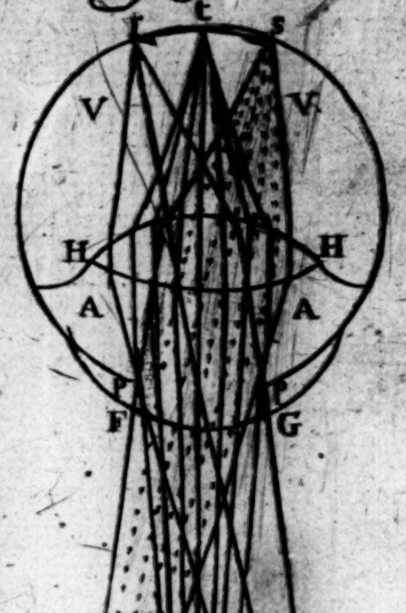


Fig. 31. P. 60.



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Form of the chriſtalline Humour, and with that the focal Diſtance of the Rays. Thus, when the Object we view is far off, and the Rays fall upon the Pupil with a very ſmall Degree of Divergency, we contract the *Ligamenta Ciliaria*, which being concave towards the vitreous Humour, do thereby compreſs it more than otherwiſe they would do; by this means it is made to preſs harder upon the back Side of the chriſtalline Humour, which is thereby rendered flatter; and ſo the Rays proceed farther before they meet in a *Focus* than otherwiſe they would have done. Add to this, that we dilate the Pupils of our Eyes (unleſs in Caſes where the Light is ſo ſtrong that it offends the Eye) and thereby admit Rays into them, that are more diverging than thoſe which would otherwiſe enter. And when the Rays come from an Object that is very near, and therefore diverge too much to be collected into their reſpective *Foci* upon the *Retina*, by relaxing the *Ligamenta Ciliaria* we give the Chriſtalline a more convex Form, by which means the Rays are made to ſuffer a proportionably greater Degree of Refraction in paſſing through it *.

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* Some Philoſophers are of Opinion, that we do this by a Power of altering the Form of the Eye; and others, by removing the Chriſtalline forwards or backwards as Occaſion requires; but neither of theſe Opinions is probable; for the Coats of the Eye are too hard, eſpecially in ſome Animals, for the firſt; and as to moving the Chriſtalline out of its Place, the Cavities of the Eye ſeem to be too well filled with the other Humours to admit of ſuch Removal.

And beſides this, by contracting the Pupils of our Eyes, we exclude the more diverging Rays, and admit only ſuch as are more eaſily refracted into their reſpective *Foci* *. But Viſion is not diſtinct at all Diſtances, for our Power of contracting and relaxing the *Ligamenta Ciliaria* is alſo circumſcribed within certain Limits.

The nearer an Object is placed to the Eye, the greater is the Image of it upon the *Retina*. Becauſe the Pencils flowing from the extreme Parts of the Object when near, make a larger Angle with each other in the Pupil where they croſs, than the ſame Pencils do when the Object is placed farther off. Thus AB (*Fig.* 33.) the Image of the Object CD, far exceeds EF that of the ſame Object GH, placed at a greater Diſtance from the Eye, as is evident from Inſpection of the Figure.

In thoſe Eyes where the *Tunica Cornea* is very protuberant and convex, the Rays of Light ſuffer a very conſiderable Refraction at their Entrance into the aqueous Humour, and are therefore collected to a *Focus* before they fall upon the *Retina*, unleſs the Object be placed very near, ſo that the Rays which enter the Eye, may have a conſiderable Degree of Divergency. People that have ſuch Eyes, are ſaid to be *purblind.* Now the nearer an Object is placed to the Eye, the greater is the Image of it therein, as explained above; theſe People therefore can ſee much ſmaller Objects than others, as ſeeing much nearer ones with the ſame Diſtinctneſs. And their Sight continues good longer than that of other People, becauſe the *Tunica Cornea* of their Eyes, as they grow old, becomes plainer, for Want of that Redundancy of Humours with which they were filled before.

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* Accordingly it is obſervable, that if we make a ſmall Hole with the Point of a Needle through a Piece of Paper, and apply that Hole cloſe to the Eye, making Uſe of it, as it were, inſtead of a Pupil, we ſhall be able to ſee an Object diſtinctly through it, though the Object be placed within half an Inch of the Eye.

On the contrary, old Men having the *Cornea* of their Eyes too flat for want of a ſufficient Quantity of the aqueous Humour to fill them out, if the Rays diverge too much before they enter the Eye, they cannot be brought to a *Focus* before they reach the *Retina*; on which Account thoſe People cannot ſee diſtinctly, unleſs the Object be ſituated at a greater Diſtance from the Eye, than is required for thoſe whoſe Eyes are of a due Form.

Since the Images of the Objects we look at are inverted in the Eye, it may be thought the Objects themſelves ought to appear ſo; but it muſt be conſidered, that there is no *natural* Connection between the *Idea* in our Mind, and the Image upon the *Retina*; we find by Experience, that when ſuch an *Idea* is excited in our Mind, ſuch an Object ſtands before us in ſuch a Poſition and of ſuch a Form; when ever therefore the like *Idea* is excited again, we conclude there is a like Cauſe of it. For it is found by Obſervation, that People who have been born blind, and have afterwards received their Sight, have had no Information from their Eyes at firſt, concerning the particular Situation or Form of Bodies; but have been obliged to ſtay till Experience has taught them what Figures and Situation of Bodies correſpond to ſuch and ſuch Senſations in the Mind *.

In like Manner it is from Experience that an Object appears ſingle, though there be an Image of it in each Eye; for after we find, that its Place, according to the Repreſentation of it in each Eye, is the ſame, we neceſſarily conceive it to be but one. The Manner how we come to find this, ſeems to be as follows: There is one Part of the *Retina* upon which when the Image falls, the Object appears brighter and more diſtinct, than when it falls upon any other, as is evident becauſe we always ſee one Part of an Object with greater Diſtinctneſs than any of the reſt. This Point I ſhall hereafter call *the Point of diſtinct Viſion.* This naturally leads us to turn our Eyes ſo, that the Object may be ſituated directly oppoſite to this Point. And this Action of ours is that which has given Riſe to thoſe imaginary Lines, which are ſuppoſed to paſs directly through the Eye and to terminate in the Object we view, and are commonly called the *optic Axes*. We therefore turn our Eyes ſo that the Object may appear in thoſe Lines. Therefore ſince theſe Lines concur at the Object, when we indeavour to view it with Diſtinctneſs, each Eye affords us an *Idea* of the Object in the ſame Place, from whence it neceſſarily appears but one *.

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* See Mr. *Cheſſelden*'s Obſervations on a young Gentleman couched by him at the Age of 13 Years. Philoſoph. Tranſact. No. 402.

When-

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* There are other Methods of accounting for theſe two laſt *Phænomena*, ſome of which, perhaps, the Reader may think more plauſible; for the Connection between the Image on the *Retina* and the Idea in the Mind, being purely *metaphyſical*, we can never hope to arrive at any Certainty in this Matter.

Some are of Opinion, that we judge thoſe Rays which paint the uppermoſt Part of the Image in the Eye to proceed from the lowermoſt Part of the Object, becauſe they ſtrike upon the *Retina*, as coming from that Part; and that we conclude from hence that the Object is erect, though the Image be inverted; as if the Direction wherewith the Rays ſtrike the *Retina*, informed the Mind which Way they came. This Solution ſerves alſo to explicate the *Phænomenon* of ſeeing but one Object with both Eyes; for as the Mind is informed by the Direction with which the Rays ſtrike the *Retina* of the Place from whence they come; therefore when it appears that they enter each Eye as from the ſame Place, the Object neceſſarily ſeems to be but one; becauſe we cant ſuppoſe two to exiſt in the ſame Place at the ſame Time. But according to this Doctrine no Object could ever appear double; becauſe wherever it be ſituated, the Rays cannot affect the *Retina* otherwiſe than if they came from it alone.

Some have been ſo abſurd as to embrace an Opinion, the firſt Author of which was *Gaſſendus*, that we ſee one and the ſame Point of an Object only with one Eye at a Time, (*otiante alio*, as they expreſs it) while the other does nothing. Vid. *Gaſſendi* Epiſtol. de Magnitud. Solis. Or *Tacquet*. Optic. Lib. I. Prop. 2.

Some imagining that the *optic* Nerves conſiſt of a Bundle of ſmall ones wrapped up in one common Tegument, are of Opinion, that ſuch as lie upon the *Retina* at equal Diſtances from the Point of diſtinct Viſion, and on the ſame Side of it in each Eye, are con-

nected

Whenever the Eyes are ſo ſituated with reſpect to an Object, that the ſame Part of the *Retina* in each Eye is affected by the Rays that flow from it, which are wont to be affected when two Objects are placed before the Eyes, the Mind receiving no Information from without, but by the Impulſes of the Rays upon the *Retina*, judges that there are two Objects. Thus let A, B (*Fig.* 34.) repreſent two Eyes whoſe *optic Axes* are directed to the Point C, and let E be an Object on one Side the Point C, and F an Object on the other. Now Objects thus ſituated muſt appear ſeparate, otherwiſe every time we viewed an Object we muſt imagine all the different Points in its Surface to be but one, which is contrary both to Reaſon and daily Experience. In this Caſe, the Point *d* in each Eye will be affected by the Rays which flow from thoſe Objects; but ſo it will, if a ſingle Object be placed at D, and therefore for the Reaſon given above, an Object in that Situation ſhall appear as the two ſeparate ones E and F, that is, double. A-

nected together in one, before they terminate in the Brain; and ſo whether one or both are affected, only one *Idea* is excited in the Mind. *Graveſand* confirms this Opinion by aſſerting, that in all Animals which look at the ſame Object with both Eyes, the optic Nerves concur before they enter the Brain; and that in ſuch as look at one Object with one Eye, and at a different one with the other, they are ſeparate all the Way.

Others, with *Briggius* (ſee his *Opththalmog.* Chap. 11.) do not contend, that the forementioned correſponding Parts of the *optic* Nerves are connected before they terminate in the Brain; but that they are of an equal Tenſion, and therefore excite the ſame Senſation in the Mind.

gain, let there be an Object placed without the *optic Axes* as at G, Rays flowing from this will affect the ſame Part in each Eye, as if there were two diſtinct Objects, *viz.* one at E and the other at H, this therefore will alſo appear double. Farther, as the Objects D and G are ſituated in this Figure, if both are attended to at the ſame Time (the *optic Axes* being ſtill directed to the ſame Point C) they will appear as three, being ſituated oppoſite to the three Points F, E, and H. And what has been ſaid of the Appearance of the Objects D and G as they are ſituated in this Figure, may be applied to their Appearance as they are placed in the next, where they are repreſented as being beyond the Line HF. So that wherever an Object is placed, provided it be nearer to the Eyes than the Point where the *optic Axes* concur, or farther from them, it appears double.

There is one Part of the *Retina* of each Eye, upon which when the Image falls, the Object cannot be ſeen at all with that Eye; the Proof of this we have from the following Experiment. Fix two Objects upon a Wall, of ſuch Bigneſs that each may hide a ſquare Inch of it, or thereabouts, and at the Diſtance of about a foot or two from each other, and go back from the Wall about three times that Diſtance; then ſhutting the left Eye, look at the left Object with the right one, and while the right Eye is in that Poſition, the right Object will not be ſeen. So if in that Station we look at the right Object with only the left Eye, the left Object will diſappear. The Reaſon of this is ſuppoſed to be, that the Image then falls upon the blood-veſſels of the *optic* Nerve, from which no Senſation is conveyed to the Brain.

The Angle comprehended between the Rays which flow from the extreme Parts of the Object, and croſs in the Pupil, is called the *optic Angle.*

'Tis by means of this Angle that we are able to form ſome Judgment of the Magnitude of an Object; becauſe the larger this is, the larger is the Image upon the *Retina*, that is, a greater Portion of it is affected by the Rays which flow from that Object. But this is not ſufficient alone, becauſe different Objects at different Diſtances from the Eye, may ſubtend equal Angles at the Pupil. We ought therefore to know alſo the Diſtance of the Object.

This, if the Object be very near, we are able to form a tolerable Judgment of, by the Degree of Divergency, wherein the Rays which flow from the ſame Point of the Object enter the Eye; becauſe we find it neceſſary to adapt the Eye accordingly, in order to bring them to a *Focus* upon the *Retina.*

But when the Object is at a greater Diſtance from us, a conſiderable Variation in the Diſtance of it makes but a very ſmall one in the

Divergency of thoſe Rays, and therefore this Rule of Judging ceaſes to be of Uſe. The only Expedient then is, the Angle comprehended between the *optic Axes* at the Object to which they are directed, or which is the ſame Thing, the Poſition of the Eyes with reſpect to each other when they view the Object *. But in very large Diſtances this Poſition varies ſo little, that it is alſo of no Uſe; in which Caſe we make the beſt Judgment we can from the Brightneſs, Diſtinctneſs, and apparent Magnitude of the Object, and likewiſe from its Situation with reſpect to others which are interpoſed †.

When we are unable to judge rightly concerning the Diſtance of an Object, we conceive it greater, the farther we imagine it to be from us, and *vice verſâ*; becauſe it requires a larger Object to exhibit the ſame Image upon the *Retina*, when it is ſituated at a great Diſtance, than when near. Thus we imagine the Sun and Moon to be farther off, when they are in the *Horizon*, than when they are near the *Meridian*, and accordingly think them proportionably larger in one Situation than in the other, though they are found to exhibit the ſame Image upon the *Retina* in both Caſes [*].

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* That the Poſition of the *optic Axes* is a Means whereby we judge of Diſtances, is evident from hence, *viz.* that they who have loſt the Sight of one Eye, find it much more difficult to eſtimate the Diſtances of Objects, than they did, when they had the Uſe of both.

† We have a remarkable Inſtance of the Error of our Judgment concerning the Diſtances of very remote Bodies, in that we look upon the Sun, Moon, and Stars to be all at the ſame Diſtance, wheras ſome of them are a thouſand Times farther from us than others.

We are never able to ſee very diſtant Objects with Diſtinctneſs; this is not ſolely owing to the Pupil's not receiving into it a ſufficient Number of Rays for that Purpoſe, or becauſe they are not collected into *Foci* upon the *Retina*, but becauſe the Object being very far off, the Rays which flow from Points of the Object that are contiguous, fall too near each other upon the *Retina* to excite diſtinct Senſations in the Mind, ſo that the *Idea* of the Whole is confuſed.

## CHAP. VII.

## *Of the Appearance of* Objects *ſeen through* Media *of different Forms.*

THat what we ſhall ſay upon the Subject of this Chapter may more readily be underſtood, we ſhall premiſe the five following Particulars; which are all compriſed in the foregoing Chapter, or follow immediately from what has been there laid down, *viz.*

1. That as each Point of an Object when viewed by the naked Eye, appears in its pro-

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[*] **See the Diſſertation on the *horizontal* Moon, annexed to this Part.**

per

Part III. Plate V. Pag. 72

Fig. 32. P. 61.

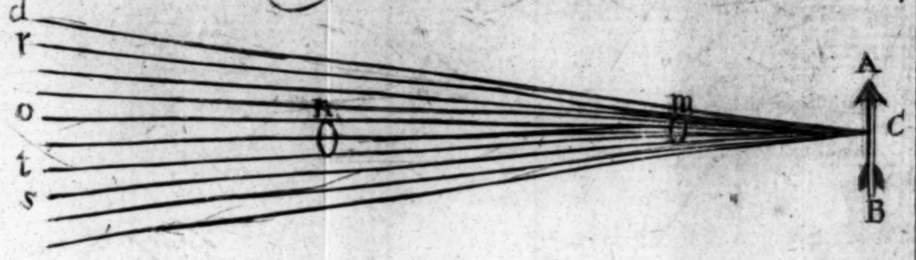


Fig. 33. P. 64.

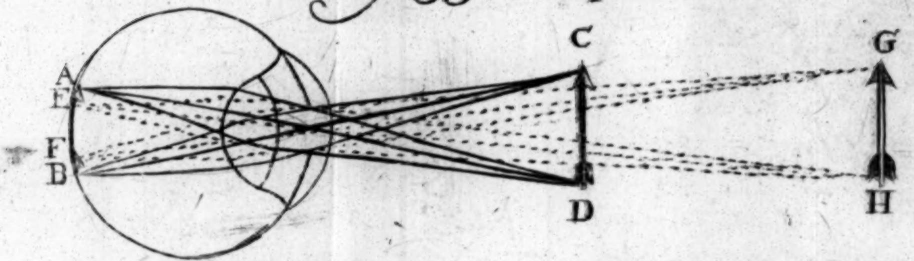


Fig. 34. P. 68.

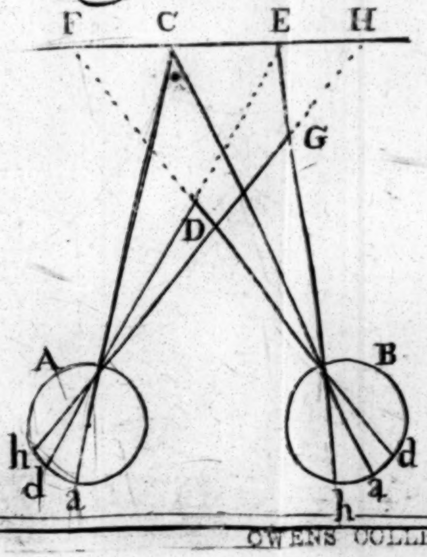
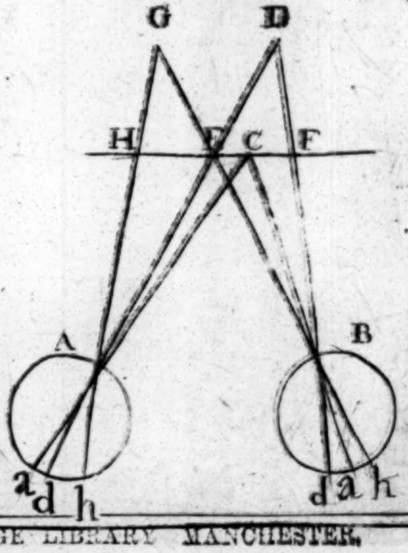


Fig. 35. P. 69.



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per Place; and as that Place is always to be found in the Line in which the *Axis* of a Pencil of Rays flowing from it, enters the Eye, we from hence acquire an Habit of judging the Point ſituated in that Line; and becauſe the Mind is unacquainted with what Refractions the Rays ſuffer before they enter the Eye, therefore, in Caſes where they are diverted from their natural Courſe by paſſing through any *Medium*, it judges the Point to be in that Line produced back in which the *Axis* of a Pencil of Rays flowing from it is ſituated the Inſtant they enter the Eye, and not in that it was in before Refraction. We ſhall therefore in what follows, ſuppoſe the apparent Place of an Object when ſeen through a refracting *Medium* to be ſomewhere in that Line produced back in which the *Axis* of a Pencil of Rays flowing from it proceed after they have paſſed through the *Medium.*

2. That we are able to judge, though imperfectly, of the Diſtance of an Object by the Degree of Divergency, wherein the Rays flowing from the ſame Point of the Object enter the Pupil of the Eye, in Caſes where that Divergency is conſiderable; but becauſe in what follows, it will be neceſſary to ſuppoſe an Object, when ſeen through a *Medium* whereby its apparent Diſtance is altered, to appear in ſome determinate Situation; in thoſe Caſes where the Divergency of the Rays at their Entrance

into the Eye is conſiderable, we will ſuppoſe the Object to appear where thoſe Lines which they deſcribe in entring, if produced back, would croſs each other; though it muſt not be aſſerted that this is the preciſe Diſtance; becauſe the Brightneſs, Diſtinctneſs, and apparent Magnitude of the Object, on which its apparent Diſtance in ſome Meaſure depends, will alſo ſuffer an Alteration by the Refraction of the Rays in paſſing through that *Medium*.

3. That we eſtimate the *Magnitude* of an Object by that of the *optic Angle*.

4. That Viſion is the *brighter*, the greater the Number of Rays is which enter the Pupil. And,

5. That in ſome Caſes, the *apparent Brightneſs*, *Diſtinctneſs*, and *Magnitude* of an Object are the only Means whereby our Judgment is determined in eſtimating the Diſtance of it.

Prop. I. An Object placed within a *Medium* terminated by a plain Surface on that Side which is next the Eye, if the *Medium* be denſer than that in which the Eye is (as we ſhall always ſuppoſe it to be, unleſs where the contrary is expreſſed) appears *nearer* to the Surface of the *Medium* than it is.

Thus if A be a Point of an Object placed within the *Medium* BCDE (*Fig.* 36), and A*b*, A*c* be two Rays proceeding from thence; theſe Rays paſſing out of a denſer into a rarer *Medium* will be refracted from their reſpective

Perpendiculars *bd*, *ce*, and will enter the Eye at H, ſuppoſe in the Directions *bf*, *cg*, let then theſe Lines be produced back till they meet in F; this will be the apparent Place of the Point A; and becauſe the refracted Rays *bf*, *cg* will diverge more than the incident ones A*b*, A*c* (Chap. III. Prop. 3.), it will be nearer to the Points *b* and *c*, than the Point A; and as the ſame is true of each Point in the Object, the Whole will appear to an Eye at H, *nearer* to the Surface BC than it is *.

Prop. II.

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* From hence it is, that when one End of a ſtrait Stick is put under Water, and the Stick is held in an oblique Poſition, it appears bent at the Surface of the Water; *viz.* becauſe each Point that is under Water appears nearer the Surface, and conſequently higher than it is.

From hence likewiſe it is, that an Object at the Bottom of a Veſſel may be ſeen when the Veſſel is filled with Water, though it be ſo placed with Reſpect to the Eye, that it cannot be ſeen when the Veſſel is empty. To explain this; Let ABCD (*Fig.* 37.) repreſent a Veſſel, and let E be an Object lying at the Bottom of it. This Object, when the Veſſel is empty, will not be ſeen by an Eye at F, becauſe HB the upper Part of the Veſſel will obſtruct the Ray EH; but when it is filled with Water to the Height GH, the Ray EK being refracted at the Surface of the Water into the Line KF, the Eye at F ſhall ſee the Object by Means of that.

In like Manner, an Object ſituated in the Horizon appears above its true Place, upon Account of the Refraction of the Rays which proceed from it in their Paſſage through the *Atmoſphere* of the Earth. For firſt, if the Object be ſituated beyond the Limits of the *Atmoſphere*, its Rays in entring it will be refracted towards the Perpendicular, that is, towards a Line drawn from the Point where they enter, to the Center of the Earth which is the Center of the *Atmoſphere*, and as they paſs on they will be continually refracted the ſame Way, becauſe they are all along entring a denſer Part, the Center of whoſe Convexity is ſtill the ſame Point; upon which Account the Line they deſcribe will be a Curve bending

Prop. II. An Object ſeen through a *Medium* terminated by plain and parallel Surfaces, appears *nearer*, *brighter*, and *larger*, than with the naked Eye.

For Inſtance, let AB (*Fig.* 38.) be the Object, CDEF the *Medium*, and GH the Pupil of an Eye, which is here drawn large to prevent Confuſion in the Figure. And 1ſt let RK, RL be two Rays proceeding from the Point R, and entring the denſer *Medium* at K and L; theſe Rays will here by Refraction be made to diverge leſs (Chap. III. Prop. 2.) and to pro-

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ing downwards; and therefore none of the Rays that come from that Object can enter an Eye upon the Surface of the Earth, except what enter the *Atmoſphere* higher than they need to do, if they could come in a right Line from the Object; conſequently the Object muſt appear above its proper Place. Secondly, if the Object be placed within the *Atmoſphere*, the Caſe is ſtill the ſame; for the Rays which flow from it muſt continually enter a denſer *Medium* whoſe Center is below the Eye, and therefore being refracted towards the Center, that is, downwards as before, thoſe which enter the Eye muſt neceſſarily proceed as from ſome Point above the Object, wherefore the Object will appear above its proper Place.

From hence it is, that the Sun, Moon, and Stars appear above the *Horizon*, when they are juſt below it, and higher than they ought to do, when they are above it: Likewiſe diſtant Hills, Trees, &c. ſeem to be higher than they are.

Farther, the lower theſe Objects are in the *Horizon*, the greater is the Obliquity with which the Rays which flow from them, enter the *Atmoſphere*, or paſs from the rarer into the denſer Parts of it, and therefore they appear to be the more elevated by Refraction; upon which Account the lower Parts of them are *apparently* more elevated than the other. This makes them ſeem ſhorter than they are; as is evident in the Sun and Moon, which appear of an oval Form when they are in the *Horizon*, their *horizontal* Diameters appearing of the ſame Length they would do if the Rays ſuffer'd no Refraction, while their *vertical* ones are ſhortened thereby.

ceed afterwards ſuppoſe in the Lines K*a* L*b*; at *a* and *b* where they paſs out of the denſer *Medium*, they will be as much refracted the contrary Way, proceeding in the Lines *ac*, *bd*, parallel to their firſt Directions (ſee Chap. IV.); produce theſe Lines back till they meet in *e*, this will be the apparent Place of the Point R, and 'tis evident from the Figure that it muſt be nearer the Eye than that Point; and becauſe the ſame is true of all other Pencils flowing from the Object AB, the Whole will be ſeen in the Situation *fg*, *nearer* to the Eye than the Line AB. 2d. As the Rays RK, RL would not have entered the Eye, but have paſſed by it in the Directions K*r*, L*t*, had they not been refracted in paſſing through the *Medium*, the Object appears *brighter*. 3d. The Rays A*h*, B*i*, will be refracted at *h* and *i* into the leſs converging Lines *hk*, *il*, and at the other Surface into *k*M, *l*M parallel to A*h* and B*i* produced (ſee Chap. IV.), ſo that the Extremities of the Object will appear in the Lines M*k*, M*l* produced, *viz.* in *f* and *g*, and under as large an Angle *f*M*g*, as the Angle A*q*B under which an Eye at *q* would have ſeen it, had there been no *Medium* interpoſed to refract the Rays; and therefore it appears *larger* to the Eye at GH, being ſeen through the interpoſed *Medium*, than otherwiſe it would have done. But it is here to be obſerved, that the nearer the Point *e* appears to the Eye on

Account of the Refraction of the Rays RK, RL, the ſhorter is the Image *fg*, becauſe it is terminated by the Lines M*f* and M*g*, upon which Account the Object is made to appear leſs; and therefore the apparent Magnitude of an Object is not much augmented by being ſeen through a *Medium* of this Form.

Farther it is apparent from the Figure, that the Effect of a *Medium* of this Form depends wholly upon its Thickneſs; for the Diſtance between the Lines R*r* and *ec*, and conſequently the Diſtance between the Points *e* and R depends upon the Length of the Line K*a*: Again, the Diſtance between the Lines AM and *f*M, depends on the Length of the Line *bk*; but both K*a* and *bk* depend on the Diſtance between the Surfaces CE and DF, and therefore the Effect of this *Medium* depends upon its Thickneſs.

Prop. III. An Object ſeen through a convex *Lens*, appears *larger*, *brighter*, and *more diſtant*, than with the naked Eye.

To illuſtrate this, let AB (*Fig.* 39.) be the Object, CD the *Lens*, and EF the Eye. 1. From A and B the Extremities of the Object draw the right Lines AY*r*, BX*r* croſſing each other in the Pupil of the Eye; the Angle A*r*B comprehended between theſe Lines, is the Angle under which the Object would be ſeen with the naked Eye. But by the Interpoſition of a *Lens* of this Form, whoſe Property it is to render converging Rays more ſo (ſee Chap. IV.) the Rays AY and BX will be made to croſs each other before they reach the Pupil. Therefore the Eye at E, will not perceive the Extremities of the Object by means of theſe Rays (for they will paſs it without entring), but by ſome others which muſt fall without the Points Y and X, or between them; but if they fall between them, they will be made to concur ſooner than they themſelves would have done, and therefore if the Extremities of the Object could not be ſeen by them, it will much leſs be ſeen by theſe. It remains therefore, that the Rays which will enter the Eye from the Points A and B after Refraction, muſt fall upon the *Lens* without the Points Y and X; let then the Rays AO and BP be ſuch. Theſe after Refraction entring the Eye at *r*, the Extremities of the Object will be ſeen in the Lines *r*Q, *r*T produced, and under the *optic Angle* Q*r*T which is larger than A*r*B, and therefore the *apparent Magnitude of the Object will be increaſed*. 2. Let GHI be a Pencil of Rays flowing from the Point G; as it is the Property of this *Lens* to render diverging Rays leſs diverging, parallel or converging (ſee Chap. IV.), it is evident, that ſome of thoſe Rays which would proceed on to M and N and miſs the Eye, were they to ſuffer no Refraction in paſſing through the *Lens*, will now enter it; by which means the Object will appear *brighter*. 3. As to the apparent Diſtance of the Object, that will vary according to the Situation of it with reſpect to the *Focus* of parallel Rays of the *Lens*. 1. Then, let us ſuppoſe the Object placed ſo much nearer the *Lens* than its *Focus* of parallel Rays, that the refracted Rays KE and LF though rendered leſs diverging by paſſing through it, may yet have a conſiderable Degree of Divergency, ſo that we may be able to form a Judgment of the Diſtance of the Object thereby. In this Caſe, the Object ought to appear where EK, FL produced back concur, which, becauſe they diverge leſs than the Rays GH, GI, will be beyond G, that is, at a greater Diſtance from the *Lens* than the Object is. But becauſe both the Brightneſs and Magnitude of the Object will at the ſame Time be augmented, Prejudice will not permit us to judge it quite ſo far off as the Point where thoſe Lines meet, but ſomewhere between that Point and its proper Place. 2. Let the Object be placed in the *Focus* of parallel Rays, then will the Rays KE and LF become parallel ( ſee Chap. IV. ) and though in this Caſe the Object would appear at an immenſe Diſtance, if that Diſtance were to be judged of by the Direction of the Rays KE and LF, yet upon Account of the Brightneſs and Magnitude of it, we ſhall not think it much *farther* from us, than if it were ſeen by the naked Eye. 3. If the Object be ſituated beyond the *Focus* of parallel Rays, as in AB (*Fig.* 40), the Rays flowing from thence and falling upon the *Lens*

CD, will be collected into their reſpective *Foci* at *a* and *b*, and the intermediate Points *m*, *n*, *&c.* and will there form an Image of the Object AB; and after croſſing each other in the ſeveral Points of it, as expreſſed in the Figure, will paſs on diverging as from a real Object. Now if an Eye be ſituated at *c*, where A*c*, B*c*, Rays proceeding from the extreme Points of the Object, make not a much larger Angle A*c*B, than they would do if there were no *Lens* interpoſed; and the Rays belonging to the ſame Pencil do not converge ſo much as thoſe the Eye would receive, if it were placed nearer to *a* or *b*, the Object upon theſe Accounts appearing very little larger or brighter than with the naked Eye, is ſeen nearly in its proper Place; but if the Eye recedes a little Way towards *ab*, the Object then appearing both brighter and larger, ſeems to approach the *Lens* *: which is an evident Proof of what has been ſo often aſſerted, *viz.* that we judge of the Diſtance of an Object in ſome Meaſure by its Brightneſs and Magnitude [*]; for the Rays converge the more the farther the Eye recedes from the *Lens*; and therefore if we judged of the Diſtance of the Object by the Direction of the Rays which flow from it, we ought in this Caſe to conceive it at a greater Diſtance, than when the Rays were parallel, or diverged at their Entrance into the Eye.

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* That the Object ſhould ſeem to approach the *Lens* in this Caſe, was a Difficulty that exceedingly puzzled the learned *Barrow*, and which he pronounces inſuperable, and not to be accounted for by any Theory we have of Viſion. *Molineux* alſo leaves it to the Solution of others, as that which will be inexplicable, till a more intimate Knowledge of the viſive Faculty, as he expreſſes it, be obtained by Mortals.

They imagined, that ſeeing an Object appears *farther off*, the *leſs* the Rays *diverge* which fall upon the Eye; if they ſhould proceed *parallel* to each other, it ought to appear *exceedingly remote*, and if they ſhould *converge*, it ſhould then appear *more diſtant ſtill*: The Reaſon of this was, becauſe they looked upon the apparent Place of an Object, as owing to the Direction of the Rays whatever it was, and not at all to its apparent Magnitude or Splendour.

Prop. IV. If an Object be placed farther from a convex *Lens*, than its *Focus* of parallel Rays, and the Eye be ſituated farther from it on the other Side, than the Place where the Rays of the ſeveral Pencils are collected into their reſpective *Foci*, the Object appears *inverted*, and *pendulous in the Air*, between the Eye and the *Lens*.

To explain this, let AB (*Fig.* 40.) repreſent the Object, CD the *Lens*, and let the Rays of the Pencil ACD be collected in *a*, and thoſe of BCD in *b*, forming there an inverted Image of the Object AB, and let the Eye be placed in F: 'Tis apparent from the Figure, that ſome of the refracted Rays which paſs through each Point of the Image, will enter the Eye as from a real Object in that Place, and therefore the Object AB will appear there, as the Propoſition aſſerts. But we are ſo little accuſtomed to ſee Objects in this Manner, that it is very difficult to perceive the Image with one Eye; but if both Eyes are ſituated in ſuch a Manner, that Rays flowing from each Point of the Image may enter both, as at G and H, and we direct our optic Axes to the Image, 'tis eaſy to be perceived.

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[*] Perhaps it may proceed from our judging of the Diſtance of an Object in ſome Meaſure by its Magnitude, that that Deception of Sight commonly obſerved by Travellers may ariſe; *viz.* that upon the firſt appearing of a Building larger than uſual, as a *Cathedral* Church, or the like, it generally ſeems nearer to them, than they afterwards find it to be.

If the Eye be ſituated in *a* or *b*, or very near them on either Side, the Object appears exceedingly confuſed, *viz.* if at *d*, the Rays which proceed from the ſame Point of the Object converge ſo very much, and if at *e*, they diverge ſo much, that they cannot be collected together upon the *Retina*; but fall upon it as if they were the Axes of ſo many diſtinct Pencils coming through every Point of the *Lens*; wherefore little more than one ſingle Point of the Object is ſeen at a Time, and that appears all over the *Lens*; from whence nothing but Confuſion ariſes.

If the *Lens* be ſo large that both Eyes may be applied to it, as in *h* and *k*, the Object will appear double; for 'tis evident from the *Figure*, that the Rays which enter the Eye at *h* from either Extremity of the Object A or B, do not proceed as from the ſame Point with that from whence thoſe which enter the other at *k* ſeem to flow; the Mind therefore is here deceived, and looks upon the Object as ſituated in two different Places, and therefore judges it to be double.

Prop. V. An Object ſeen through a concave *Lens* appears *nearer*, *ſmaller*, and leſs *bright*, than with the naked Eye.

Thus, let AB (*Fig.* 41.) be the Object, CD the Pupil of an Eye, and EF the *Lens*. Now, as it is the Property of a *Lens* of this Form, to render diverging Rays more ſo, and converging ones leſs ſo, the diverging Rays GH, GI, proceeding from the Point G, will be made to diverge more, and ſo to enter the Eye as from ſome nearer Point *g*; and the Rays AH, BI, which converge, will be made to converge leſs, and to enter the Eye as from the Points *a* and *b*; wherefore the Object will appear in the Situation *agb*, *leſs* and *nearer* than without the *Lens*. Farther, as the Rays which proceed from G, are rendered more diverging, ſome of them will be made to paſs by the Pupil of the Eye which otherwiſe would have entered it, and therefore each Point of the Object will appear *leſs bright* *.

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* From what has been obſerved about the Properties of convex and concave *Lenſes*, we may ſee the Reaſon why the former Sort are made Uſe of by *old People* to help their Sight; and the latter by thoſe who are *purblind*. *Old People*, as was obſerved before, having the *Tunica Cornea* of their Eyes too flat, require that the Object be placed at a greater Diſtance from them, than other People whoſe Eyes are of a juſt Form, that the Rays which enter the Pupils of their Eyes from the ſame Point of the Object, may not diverge

Prop. VI. An Object ſeen through a *polygonous* Glaſs, that is, ſuch as is terminated by ſeveral plain Surfaces, is *multiplied* thereby.

For Inſtance, let A (*Fig.* 42.) be an Object, and BC a polygonous Glaſs terminated by the plain Surfaces BD, DE, *&c.* and let the Situation of the Eye F be ſuch, that the Rays AB being refracted in paſſing through the Glaſs, may enter it in the Direction BF, and the Rays AC in the Direction CF. Then will the Eye by means of the former, ſee the Object in G, and by the latter in H; and by means of the Rays AI, the Object will appear alſo in its proper Situation A.

Thus much for the Principles of *Dioptics*, and the Solution of ſome obvious *Phænomena* which tend to confirm the ſame: Thoſe which yet remain to be accounted for, ſhall according to the Method we have hitherto obſerved, be treated of in the Diſſertations of this Part.

verge too much. Now a convex *Lens* makes thoſe Rays diverge leſs, as they would naturally do if the Object was placed farther off. Again, thoſe who are *purblind*, having the *Tunica Cornea* too protuberant, require ſuch a *Lens* as may render thoſe Rays more diverging, leſt they ſhould be collected into their reſpective *Foci* before they fall upon the *Retina*; and therefore *Lenſes* of the concave Sort are of Uſe to them.

## DISSERTATION I.

## *Of the* Horizontal Moon.

THE *Phænomenon* of the *horizontal* Moon is this: when the Moon is juſt above the Surface of the Earth, either immediately after ſhe is riſen, or juſt before ſhe ſets, ſhe appears four or five Times greater in Diameter, than when ſhe is in her Meridian Altitude: And yet her apparent Diameter, if taken by an Inſtrument, is found to ſubtend the ſame Angle in either Situation *.

The Moon's apparent Diameter being found to ſubtend the ſame Angle, whether ſhe be in the Horizon or Meridian, it is evident the Image of her projected upon the *Retina* of an Eye, is of the ſame Dimenſions in either Caſe; and therefore that ſhe ſhould appear of a different Magnitude in one Situation from what

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* What is ſaid here of the Moon's Diameter, as taken by an Inſtrument, muſt be underſtood of her *horizontal* Diameter, and not of her *vertical* one, for the Length of this is diminiſhed by Refraction (as explained Chap. VII. Note the firſt), and therefore if it be taken by an Inſtrument, it will not be found to ſubtend the ſame Angle in the Horizon as in the Meridian: But notwithſtanding this, it appears longer to the naked Eye when in the former, than in the latter Situation, as well as the horizontal Diameter.

*Fig. 36. P. 74.*

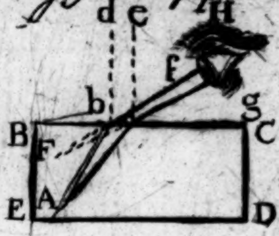


*Fig. 37. P. 75.*

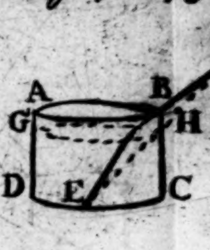


*Fig. 38. P. 76.*

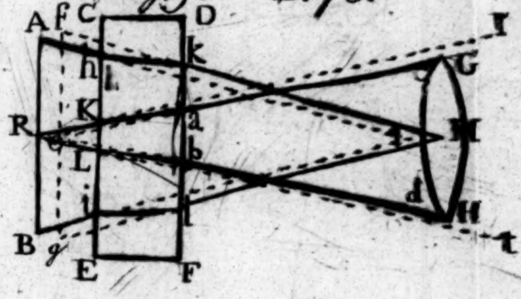


*Fig. 39. P. 78.*

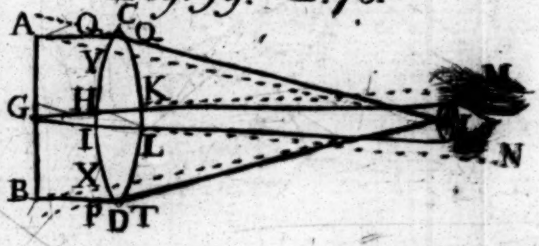


*Fig. 40. P. 82.*

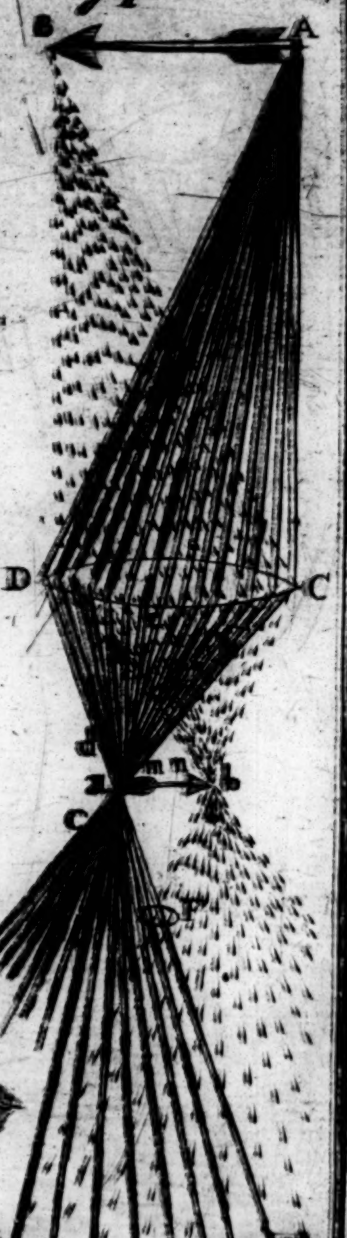


*Fig. 41. P. 84.*

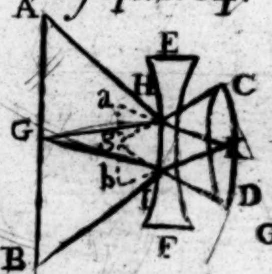
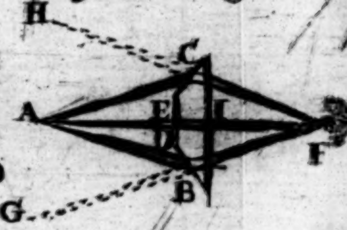


*Fig. 42. P. 85.*



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ſhe does in the other, has always been Matter of great Speculation among the *Connoiſſeurs* both in *Optics* and *Aſtronomy*. *Des Cartes* was of Opinion, that we think the Moon larger when ſhe is in the Horizon, than when ſhe is in the Meridian, becauſe in the former Caſe by comparing her Diſtance with that of interpoſed Objects, we imagine it greater than when ſhe is elevated: And that as we judge her Diſtance greater in that Situation, we of Courſe think her Diameter longer, becauſe it ſubtends the ſame Angle in either Caſe. But more of this by and by, when we come to the Explication Dr. *Wallis* has given of this Matter.

*Gaſſendus* was of Opinion, that becauſe the Moon appears leſs bright when in the Horizon than in the Meridian, we view her in the former Situation with a larger Pupil, than we do in the latter; and from thence he concludes, that the Image of her upon the *Retina* muſt be larger. But this is contrary to the Laws of *Optics*; for if the refractive Power of the Humours of the Eye collects the Rays of the ſeveral Pencils into their reſpective *Foci* upon the *Retina* (and there is no Reaſon to ſuppoſe the contrary in this Caſe) the Breadth of the Pupil makes no Alteration in the Magnitude of the Image; becauſe the Situation of thoſe *Foci* is determined by the *Axes* of the ſeveral Pencils, which croſſing each other in the Center of the Pupil (as was ſhewn Chap. VI. pag. 61. in the Note) paſs on to the ſame Points of the *Retina*, whether the Pupil be broad or narrow.

*Molineux* in the Philoſophical Tranſactions No. 187. tells us of a certain *French Abbé*, that revived the forementioned Suppoſition of *Gaſſendus*, and adding two others of his own, endeavour'd to account for this *Phænomenon*. His Suppoſitions were theſe, *viz.* " That this " contracting and enlarging the Pupil (ſup- " poſed by *Gaſſendus*) cauſeth a different " Shape in the Eye; an open Pupil making " the Chriſtalline flatter, and the Eye longer, " and the narrower Pupil ſhortning the Eye, " and making the Chriſtalline Humour more " convex. The firſt attends our looking at " Objects that are remote, or which we think " ſo, the latter accompanies the viewing Ob- " jects nigh at Hand. Likewiſe an open Pu- " pil and flat Chriſtalline attends Objects of " a more ſedate Light, whilſt Objects of more " forcible Rays require a greater Convexity, " and narrower Pupil. From theſe Poſitions, " continues *Molineux*, the *Abbé* endeavoured " to give an Account of our *Phænomenon*, as " follows. When the Moon is nigh the Ho- " rizon, by Compariſon with interpoſed Ob- " jects, we are apt to imagine her much far- " ther from us than when more elevated, and " therefore we order our Eyes as for viewing

" an Object farther from us; that is, we ſome-
" thing enlarge the Pupil, and thereby make
" the Chriſtalline flatter: moreover the Duſ-
" kiſhneſs of the Moon in that Poſture does not
" ſo much ſtrain the Sight; and conſequently
" the Pupil will be more large, and the Chriſ-
" talline more flat; hence a larger Image ſhall
" be projected on the Fund of the Eye, and
" therefore the Moon ſhall appear larger.
" Theſe two forementioned Accidents, *viz.*
" the Moon's imaginary Diſtance and Duſ-
" kiſhneſs gradually vaniſhing as ſhe riſes, a
" different *Species* is hereby introduced in the
" Eye, and conſequently ſhe ſeems gradually
" leſs and leſs, till again ſhe approaches nigh
" the Horizon.

As to what is taken for granted in this Solution concerning a Change in the Chriſtalline Humour and Form of the Eye, upon viewing an Object in a *dusky* or faint Light, that ſeems to be very ill grounded. We know of no ſuch Connection between the Muſcles of the *Iris* and thoſe of the *Ligamenta Ciliaria*, as is neceſſary to produce this Effect. And the Coats of the Eye are not ſo pliable, as eaſily to admit of an Alteration in their Form *. Could the Author have made good his other Suppoſition, *viz. That by Compariſon with*

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* See what has been obſerved concerning the Power we have of making an Alteration in the Eye, in order to ſee *diſtinctly.* (Chap. VI. pag. 62.)

*interpoſed Objects we are apt to imagine her much farther from us, than when more elevated*, he need not have had Recourſe to any other; this alone would have been ſufficient; but *hic Labor eſt.* This alone, I ſay, would have been ſufficient; for if by comparing her Diſtance with that of interpoſed Objects, we imagine it greater when ſhe is in the Horizon, than when ſhe is in the Meridian; as ſhe ſubtends an equal Angle in both Caſes, we muſt in Conſequence thereof (agreeably to *Des Cartes's* Notion above-mentioned) imagine her to be bigger in the former than in the latter; becauſe a diſtant Object cannot ſubtend the ſame Angle at the Eye that one which is nearer does, unleſs it be proportionably larger *.

The famous *Hobbs* endeavour'd at a Solution of this *Phænomenon*, but it is hardly worth mentioning: The Figure he has drawn to explain his Solution by, ſeems to have been the Occaſion of his Errour. He draws a Circle to repreſent that blue Surface commonly called the Sky, in which the heavenly Bodies ſeem to be fixed, and concentric to this, a leſſer, to repreſent the Surface of the Earth, but vaſtly too big in Proportion; ſo that a Spectator upon the Surface of this Earth, is conſiderably nearer to the upper Part of the other Circle than to the Sides of it: Wherefore an Object that ſubtends the ſame Angle at different Heights, muſt neceſſarily hide a greater Portion of that Ark when it is in the Horizon, than when it is in the Meridian; becauſe that Ark is farther behind the Object in the former than in the latter Situation; from whence he concludes that the Moon muſt appear bigger in that Situation than in the Meridian. Had he drawn his Circles in any tolerable Proportion to that which he deſigned them to repreſent, he would eaſily have ſeen his Miſtake.

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* See Chap. VI. p. 71.

A few Years ago Mr. *De Veil* publiſhed a Treatiſe upon the Subject of the horizontal Moon, which he dedicates to the Ladies of *Northampton*. If I remember right, his Solution of it was in the following Manner. 1. When an Object is placed beyond the *Focus* of parallel Rays of a convex *Lens*, the farther the Eye (ſituated on the other Side the *Lens*) receeds from it towards the *Focus* of the Rays which flow from that Object, the larger that Object appears. 2. Rays of Light flowing from the Moon, and paſſing thro' the *Atmoſphere* of the Earth, are collected into a *Focus* on the other Side of it. 3. When the Moon is in the Horizon, we are nearer to this *Focus* by almoſt a *Semidiameter* of the Earth, than when ſhe is in the Meridian: And therefore the Moon ought to appear larger when in the former than in the latter Situation.

The ſecond Propoſition in this Solution is true, but not applicable in the preſent Caſe; for unleſs we conſider the Refraction that Rays of Light which flow from the Moon, and paſs through the Atmoſphere of the Earth, ſuffer in their Emerſion, that is, while they paſs through the latter half of it, as well as that which they ſuffer in their Immerſion, or while they paſs through the former half, we ſhall find that they will not be collected into their reſpective *Foci* on the other Side the Earth, as this Gentleman imagines: Which if it can be ſhewn, his Solution falls to the Ground of Courſe; for the Refraction which the Rays ſuffer in their Emerſion is not to be taken into Conſideration, becauſe they reach the Eye of a Spectator upon the Earth as ſoon as they have paſſed through the firſt half of the Atmoſphere, when the Moon is in his Horizon; and before they have paſſed through that half, when ſhe is in his Meridian.

Let us then imagine two Rays flowing from one and the ſame Point of the Surface of the Moon, it being neceſſary in order to conſtitute a *Focus* that ſuch Rays ſhould after Refraction meet in a Point; the meeting of ſuch as flow from different Points in the ſame Surface is not ſufficient; if it were, we might then have *Foci* where we pleaſed, and that as well without refracting or reflecting Surfaces as with them. And let the firſt of thoſe Rays fall perpendicularly upon the Atmoſphere of the Earth, and be ſuppoſed to paſs through the Center of it, and let the other after Refraction paſs by the Surface of the Earth. Now the Moon's *Parallax*, that is, the Angle under which the Semidiameter of the Earth is ſeen from the Moon, being about one Degree, it is evident that theſe Rays muſt, before their Incidence upon the Earth's Atmoſphere, diverge the one from the other by ſuch an Angle. But it appears from Sir *Iſaac Newton*'s Table of Refractions publiſhed by Dr. *Halley* in the Philoſophical Tranſactions, No. 368, that when any of the Heavenly Bodies appears in the Horizon, the Rays by which it is ſeen, are refracted but by an Angle of thirty three Minutes and forty five Seconds; and therefore the Ray which we have ſuppoſed after Refraction to paſs by the Surface of the Earth, will be refracted only by ſuch an Angle. Which falling conſiderably ſhort of one Degree, the Angle by which it diverged from the perpendicular one before Refraction; it will be ſo far from being made to converge towards it thereby, that it will ſtill be in a State of Divergency from it. And therefore Rays flowing from the Moon and refracted only in their Immerſion into the Atmoſphere of the Earth, will not be collected into their reſpective *Foci* on the other Side: Which was to be ſhewn.

Dr. *Wallis* in the Philoſophical Tranſactions No. 187, gives us a Solution of the horizontal Moon (or rather an Explication of what *Des Cartes* had given before) which is as follows.

He aſcribes this *Phænomenon* to the Deception of the Imagination, and accounts for that Deception in the following Manner. He obſerves firſt, that the Imagination doth not eſtimate the Greatneſs of an Object ſeen, by the *optic Angle* only, but by this compared with the ſuppoſed Diſtance. So that if two Things are ſeen under the ſame or equal Angles, and if upon any Account whatever we apprehend one of theſe to be farther from us than the other, that which we apprehend to be farther from us, will to the Imagination appear greater. Secondly, That one great Advantage for eſtimating the apparent Diſtance of any Thing, is from the Variety of intermediate Objects between the Eye and the Thing ſeen; for then the Imagination muſt allow Room for all theſe Things.

" Now ſays he, when the Sun * or Moon is " near the Horizon, the Proſpect we have of " Hills and Vallies, Plains and Woods, *&c.* " repreſent to our Imagination a great Diſ- " tance capable of receiving all theſe. Or if " it happens that theſe interpoſed Objects are " not actually ſeen; yet having been accuſ- " tomed to ſee them, the Memory ſuggeſts to " us a View as large as is the viſible Horizon.

" But when the Sun or Moon is in an high- " er Poſition, we ſee Nothing between us and " them (unleſs perhaps ſome Clouds) and

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* For the Sun appears larger in the *Horizon*, as well as the Moon.

" therefore Nothing that can preſent to our
" Imagination ſo great a Diſtance as the other
" is. And therefore though both be ſeen under
" the ſame Angle, they do not appear (to
" the Imagination) of the ſame Bigneſs, be-
" cauſe not fanſied at the ſame Diſtance: But
" that near the Horizon is judged bigger (be-
" cauſe ſuppoſed farther off) than the ſame,
" when at a greater Altitude.

This is the Solution which has hitherto generally been received; but I am told that a more probable one will ſhortly be given by Dr. *Smith*, Plumian Profeſſor in the Univerſity at *Cambridge*, in a Treatiſe of *Optics* he is now preparing for the Preſs.

If I might be allowed to mention any Thing of my own, after theſe great *Genius*'s have given their Opinions upon this Matter, it ſhould be this, *viz.* That I have often thought, that he who would give a rational Account, why the Sun or Moon appears farther from us in the Horizon, than in the Meridian (for that is all that is requiſite towards a Solution of the Horizontal Moon, as has been already obſerved) ſhould firſt ſhow, why that apparent azure Surface we call the Sky, does not ſeem to be an entire concave *Hemiſphere*, but only a Portion of ſuch an one. For our judging the Heavens to be no more than ſuch a Portion, is undoubtedly the Cauſe why we judge both the Sun, Moon, and Stars to be farther from us when in the Horizon, than in the Meridian;

becauſe we have Nothing elſe we can refer their Places to, but that.

Now poſſibly the Cauſe why we think the Heavens of that Form, may after all be only this, *viz.* that, as the Rays which come from the upper Parts of that imaginary Surface the Sky, paſs through a leſs Portion of the *Atmoſphere*, than ſuch as come from the horizontal Parts of it, the Sky appears to us more *diſtinctly* and generally more *bright* in thoſe Parts, than in the latter; and therefore, ſince we daily obſerve that thoſe Objects which appear moſt *diſtinct*, are, generally ſuch as are *neareſt* to us; and alſo as *bright* Objects, when we have Nothing but bare Imagination to determine us in eſtimating the Diſtance of them, appear *nearer* to us than the ſame Objects when leſs ſo *, we think the upper Parts of the Sky *nearer* us than the lower. Wherefore, ſince we refer all the heavenly Bodies to this Surface, we neceſſarily imagine them farther from us, and conſequently larger, and alſo more diſtant from each other †, when near the Horizon, than when they are arrived at their meridian Altitude.

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* See what has been ſaid concerning the Brightneſs of an Object being a Means whereby it appears nearer us, under Prop. 3, of the 7th Chapter.

† The apparent horizontal Diſtance of two Stars from one another, is obſerved ~~to be greater when~~ they are in the Horizon, than in the Meridian.

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